Integrating a Free Online Service: Advertisements and Competitive Implications

Gaurav Jakhu*

Abstract

This paper examines firms' incentives to integrate free, advertising-financed online services with hardware products and their implications. Our analysis shows that service integration has a non-monotonic effect on hardware prices while reducing advertising levels. Moreover, when the quality advantage is sufficiently large, integration increases both firms' profit by enabling them to monetize different user segments. However, for small to intermediate quality advantage, service integration can make the dominant firm better off and rival firm worse when the nuisance cost of advertisement is small to intermediate, and both firms better off when the nuisance cost of advertisement is sufficiently large. Finally, service integration can benefit users but harm advertisers.

Key Words: Platform Integration; Multi-Product Firms; Platform Competition; Welfare.

*Indian Institute of Management Bangalore, India; E-mail: gaurav.jakhu@iimb.ac.in

1 Introduction

Big technology firms increasingly compete at the system level, selling both hardware products and complementary online services. A prevalent strategy is service integration, where firms make their hardware and services incompatible with rival firms' offerings, leading to a "walled garden" approach. Therefore, service integration leads to competition between firms' systems, and consumers are restricted to choosing between systems, each comprising of hardware and service of a single firm. For example, in markets for smartphones and smart displays, proprietary application stores or voice assistants are tightly integrated with devices (e.g., Apple–App Store, Huawei–AppGallery; Google Nest Hub–Google Assistant, Amazon Echo Show–Alexa).[1]

Service integration in our paper is implicitly similar to a competitive (pure) bundling/ incompatibility strategy: a firm's components are incompatible with rivals and must be consumed as a system. In the symmetric duopoly, Matutes and Regibeau (1988) showed that incompatibility increases the length of the equilibrium market boundary-intensifying price competition and decreasing both firms' profit. Building on this, Hahn and Kim (2012), and Hurkens et al. (2019) introduce cost and quality asymmetry, and find that incompatibility makes the distribution of consumers' average locations more concentrated, thus decreasing the length of the equilibrium market boundary and softening price competition. A recent paper by Adner et al. (2020) find that incompatibility can benefit both firms by differentiating revenue sources.

However, these papers do not consider the market structure illustrated in the earlier examples, i.e., a market with a paid hardware product and a free, advertising-financed online service: a two-sided market structure for services, where advertisers value users while users dislike advertisements We contribute to the theory of competitive bundling and incompatibility choice by considering a a two-sided market structure for services and addressing the following research questions: What is the impact of service integration on equilibrium prices and advertisements? Under what market conditions, service integration is a profitable strategy? Can a firm with a superior product profitably leverage its quality advantage from the product to the service dimension? How will service integration affect user welfare and advertisers' profit? To answer these questions, we develop a game-theoretic model examining competition between two firms with both having two components: a hardware product and a free online service, generating profit from both hardware sales to the single-homing users and advertising revenue from advertisements in the services, which impose a nuisance cost (disutility) on the service users. Moreover, one firm has a "quality advantage" for the product. Both firms make service integration decisions first and then set their hardware prices and advertising quantities, and finally, advertisers make adoption decisions and consumers purchase hardware products and services. If

[1] We discuss these examples in greater details later in Section 1.1.

at least one firm adopts service integration, then competition takes place between two incompatible systems, each system having a product and a service.

Under service integration, firms with access to single-homing users have market power over the multi-homing advertisers who are trying to reach these users. What is the effect of this market power over advertisers on equilibrium decisions? On the user side, the hardware prices are guided by two distinct mechanisms: the conventional mechanism, according to which service integration decreases demand elasticity leading to higher user prices, and the two-sided mechanism, which decrease prices so as to attract more users for the advertisers. In equilibrium, the price is driven down by the full amount of advertising revenue obtained. The final price crucially hinges on the strength of these two mechanisms. On the advertiser side, advertising quantities are chosen to maximize the difference between the benefit and cost (disutility imposed on users) of placing an advertisement. As a result, the equilibrium advertisements decrease because service integration results in firms internalizing the two-sided interaction between advertisers and users, reducing their advertising levels to prevent users from switching.

We next provide an intuition about the profit impact of service integration. Interestingly, service integration helps firms to strategically focus on a different component's user base, i.e., it leads to market segmentation. Unlike Adner et al. (2020), for the firm with better product quality, market share for the service becomes more important but less important for the rival firm. Moreover, it decreases products' demand elasticities because of a decrease in users' sensitivity to a price change, which dominates the change in the average location of the indifferent user, thus strengthening firms' incentives to increase prices. Also, the platform market structure of the services generates a new competitive bottleneck effect: under service integration, profits are independent of advertising revenue because either there is a full pass-through of advertising revenue to the users in the form of lower prices or firms choose zero advertisements. In contrast, under independent pricing, firms earn positive advertising revenue. Therefore, under service integration, firms cannot profit from advertising because of the competitive bottleneck effect. When the level of quality advantage is sufficiently small, service integration is not profitable for either firm. This occurs because neither firm gains a sufficient market share and the demand elasticity effect is also weak, making a possible price increase small. Together, these effects are dominated by the competitive bottleneck effect, thus decreasing firms' profit. The reverse holds when quality advantage is sufficiently large. Both firms adopt service integration as they gain sufficient market shares, and reduced demand elasticity effect dominates the competitive bottleneck effect, thus raising prices charged by both firms and benefits both firms. However, for an intermediate level of quality advantage, the effect of the integration strategy crucially depends on the nuisance cost of advertisements. For a sufficiently large nuisance cost of advertisements, the competitive bottleneck effect is weak and dominated by the reduced demand elasticity effect, thus increasing both firms' prices and profit. However, for a small to intermediate level of nuisance cost of advertisements, the competitive bottleneck effect is strong, and service integration has an asymmetric effect on firms' profit. The better quality firm gains as reduced demand elasticity effect outweighs the competitive bottleneck effect, generating higher profit. Whereas, for the inferior quality firm, the competitive bottleneck effect dominates the reduced demand elasticity effect, resulting in lower profit. Importantly, this shows that the better quality firm can profitably leverage its quality advantage to the service dimension, putting the rival at a disadvantage.

Next, we briefly discuss novel theoretical insights based on our analysis. In prior literature, incompatibility is not profitable because it can increase the length of equilibrium market boundary, which intensifies price competition (Matutes and Regibeau 1988, Hahn and Kim 2012, and Hurkens et al. 2019). However, in contrast to these studies, as discussed above, with service integration, demand becomes inelastic. Also, firms cannot profit from advertisements under service integration because of the competitive bottleneck effect. Therefore, unlike Matutes and Regibeau (1988), Hahn and Kim (2012) and Hurkens et al. (2019), price competition under service integration is intensified only when competitive bottleneck effect dominates the reduced demand elasticity effect; otherwise not. For a large quality advantage, the results in Matutes and Regibeau (1988) are reversed. Similarly, when there is a small to intermediate quality advantage and a large nuisance cost of advertisements, the findings in Matutes and Regibeau (1988), Hahn and Kim (2012) and Hurkens et al. (2019) are reversed. For this parameter space, the weak competitive bottleneck effect is dominated by reduced demand elasticity effect with service integration, which increases both firms' prices and profit. Finally, Adner et al. (2020) find that under incompatibility, the superior hardware firm would rely mainly on hardware revenues. We contribute to a better understanding of how service integration affects price and advertising competition by introducing advertising in services and user disutility from viewing advertisements. Importantly, unlike Hahn and Kim (2012), Hurkens et al. (2019) and Adner et al. (2020), the platform market structure of services identifies new conditions for profitable leverage of dominant firm's quality advantage to the service dimension.

Our analysis also provides insights into some of the dynamics that we can observe in the digital market. For a large difference in hardware functionalities, firms have a strong incentive to integrate their free services with hardware devices. This helps them to strategically differentiate their products and focus on different components' user base to obtain higher profits. In the smartphone market, Apple and Huawei have significant differences in the functionalities. Hence, both prefer a closed ecosystem approach with integration between their devices and free services such as application stores, etc. Similarly, in smart displays, Google Nest Hub's superior ecosystem integration gives it a quality advantage over Amazon Echo Show. Service integration allows Google to expand its Assistant user base, while Amazon benefits from keeping Echo Show integrated with Alexa to grow its device user base. Finally, in smart glass market, both Meta and Amazon, have tightly integrated their smart glasses with their proprietary voice assistants, which can allow Meta to increase its Assistant user base and Amazon to expand on device sales.

Regarding welfare effects, we find asymmetric effects on users and advertisers. For instance, for small to intermediate difference in product qualities, service integration increases user welfare but decreases advertisers' profit. An important policy lesson is that a blanket regulation would be ineffective and should consider the nuances of market characteristics, balancing one side's gain against the other side's loss.

The rest of the paper is organized as follows. Section 1.1 describes service integration in two-sided markets. Section 2 discusses the related literature. Section 3 presents the model. Section 4 examines the equilibrium outcomes. Section 5 discusses a few extensions to the baseline model. Section 6 discusses the theoretical contributions. Section 7 conducts a welfare analysis. Section 8 discusses managerial and policy implications and concludes. All proofs are in the appendix.

1.1 Service Integration in Two-Sided Markets: Examples

Over the last few decades, big technology firms have expanded their presence across markets and own both paid hardware products and free online services (with advertisements in it), and have widely adopted service integration.

Consider the smartphone market, where Apple and Huawei are the two main competitors. Both own smartphones and application stores. Apple offers the App Store, while Huawei provides the AppGallery. However, both firms have made their smartphones and application stores incompatible with their rival components. A similar trend of service integration is visible in the smart home market (e.g., smart display), and wearable devices market (e.g., smart glass), where major device manufacturers have tightly integrated their proprietary voice assistants with their devices, thus creating incompatible systems.

As another example, consider the the smart display market with two main competitors: Google Nest Hub and Amazon Echo Show. Both own voice assistant services to access the content through their smart displays: Google Assistant and Amazon Alexa. Moreover, like in the smartphone market, both firms maintain closed ecosystems, ensuring that their smart displays and voice assistants remain incompatible with competitors' offerings. In the smart glass market, Meta AR glasses and Amazon AR glasses are tightly integrated with their proprietary voice assistants: Meta AI and Alexa, respectively, thus resulting in an incompatibility between the two systems.[2]

[2]The European Commission has highlighted service integration in smart home and wearable device market as a significant competition concern. In its Consumer IoT Sector Inquiry 2020 report, EC has argued that "*attempts to secure exclusivity of voice assistant presence on smart devices could potentially raise competition concerns if they prevent other competing voice assistants from*

The online services illustrated in the markets mentioned above are free to use for the users, however, they include advertisements. For instance, Apple's App Store offers advertising options to the app developers to promote their apps. Likewise, Huawei offers advertising slots in its AppGallery. In the smart display market, advertisements are embedded in voice assistant (VA) interactions. For example, both Google and Amazon have started including ads in VA interactions with users through Google Assistant and Alexa, respectively (e.g., Williams 2019, Hirsch and Castillo 2018).[3] Moreover, the value of placing an advertisement for advertiser increases with the number of users consuming the service; whereas, users dislike advertisements placed in these services. In the smart display market, recent studies have highlighted negative reactions to advertisements by voice assistants (Snyder et al. 2024). Thus, these online services have a two-sided (platform) market structure.

2 Related Literature

The novel contribution of our research lies in connecting the literature on two-sided markets, with the literature on competitive bundling/incompatibility and platform envelopment, thereby generating new insights.

Platform Competition and Network Effects: We build on previous literature examining competition between firms with network effects (Katz and Shapiro 1985) and competition between advertising-financed platforms (e.g., Rochet and Tirole 2003, Anderson and Coate 2005, Armstrong and Wright 2007) by explicitly considering two different components: a hardware product and an online service, and asymmetry in product quality. Moreover, we consider services to be horizontally differentiated, with a platform market structure connecting users and advertisers. We also examine endogenous service integration decisions and leverage of market power.

Product Compatibility and Competitive Bundling: Our paper is also related to the literature on product compatibility and competitive bundling that allows for system competition between firms, and consumer demand systems made of complementary components. Matutes and Regibeau (1988) show that symmetric firms choose full compatibility between components, since it weakens the internalization of complementarity between components, raising prices compared to incompatibility. Kim and Hahn (2022) further clarifies this by identifying two distinct effects of incompatibility: a change in the length of the market boundary (which can intensify competition if it increases) and higher transportation costs (softening competition). Using this distinction, in Matutes and Regibeau (1988), incompatibility increases the length of the equilibrium

being built-in simultaneously on the devices." For instance, these competition concerns can emerge because of self-preferencing of own products and services by voice assistants, collection of user data, etc. See "Commission Staff Working Document," paragraph 2, page 112, available at https://ec.europa.eu/commission/presscorner/detail/en/ip$_2$2$_4$02.

[3] Similarly, the growth of smart glasses, and VA interactions based on it can provide a new opportunity to deliver advertisements to consumers. See."Next-Generation Advertising: Smart Glasses And AI-Driven Engagement," March 2025, Forbes, available at https://www.forbes.com/councils/next-generation-advertising-smart-glasses-and-ai-driven-engagement.

market boundary which dominates the transportation cost effect, leading to intensified price competition and lower profits. Our model differs in two key ways. First, service integration always reduces demand elasticity, as the transportation cost effect dominates the change in the length of the market boundary for any positive quality advantage level. Second, we identify a novel effect where service integration can lower prices through a full pass-through of advertising revenue to users. Thus, firms cannot profit from advertising with integration but gain advertising revenue under independent pricing. Together, these effects lead to new findings. In contrast to Matutes and Regibeau (1988), service integration can increase profits when the quality advantage is large or when quality advantage is small to intermediate and the advertisement nuisance costs are large. This occurs because the demand becomes sufficiently inelastic, which dominates the lost advertising revenue.

Few recent papers (e.g., Zhou 2017, Kim and Choi 2015) have extended the analysis of Matutes and Regibeau (1988) by examining competitive bundling under different modelling assumptions. Kim and Choi (2015) and Zhou (2017) both examine bundling in a symmetric oligopoly framework with n symmetric firms. They show that, for the duopoly case, incompatibility is not profitable because it intensifies price competition. However, when the number of firms is large enough, bundling increase prices and thus benefits firms. As described above, unlike these papers, we identify a new mechanism, based on the two-sided market structure of services to explain the profitability of service integration.

A few recent papers have also examined the role of firm dominance on competitive bundling decisions (e.g., Hahn and Kim 2012, Hurkens et al. 2019). Both Hahn and Kim (2012) and Hurkens et al. (2019) introduce firm asymmetry into Matutes and Regibeau (1988) with a single firm dominating all markets. They show that incompatibility changes the distribution of consumers' average locations: more peaked at the center and thinner at the tails compared to the distribution of locations under compatibility. As a result, incompatibility affects firms' profit by changing the average location of the indifferent consumer or the length of the equilibrium market boundary (which depends on the dominance level of the dominant firm). Under incompatibility, for a small (large) dominance level, the average location of the indifferent consumer is close to the center (tail) of the distribution, thus making demand sufficiently elastic (inelastic) and resulting in lower (higher) prices compared to the compatibility regime. Our paper setting differs from these studies as we consider a platform market structure, i.e., purely advertising financed services with consumers' aversion to advertisements. Interestingly, unlike these studies, for a positive quality advantage, service integration always results in inelastic demand because consumers' sensitivity to a price change reduces, which dominates the change in the average location of the indifferent consumer (or the length of equilibrium market boundary). It is the interaction between users and advertisers, which can reduce hardware prices under service integration because of the pass-through of advertising revenue through lower prices. As a result, for small to intermediate quality advantage and large nuisance cost of advertisement, unlike Hahn and Kim (2012) and Hurkens et al. (2019), we find that service integration increases both firms' profit because of reduced demand elasticity which increase firms' prices and dominates the lost profit from advertising. Finally, the platform market structure also provides new conditions for credible leverage of market power through service integration.

Regarding platform compatibility, the closest paper to our work is by Adner et al. (2020). However, there are significant modelling differences. First, in Adner et al. (2020), for a fraction of consumers, preferences are perfectly correlated for product and software, and hardware preferences play the dominant role. This distinction is crucial. To highlight that, when all consumers have independent preferences for product and software, Adner et al. (2020) find that incompatibility does not change the strength of users' sensitivity to a price change because hardware preferences still play the dominant role. Thus, incompatibility intensifies competition (because of user subsidization) which decreases firms' prices and profit. In contrast, in our paper, with independent consumer preference for the product and service, service integration decreases users' sensitivity to a price change, which dominates the change in the length of equilibrium market boundary, thus reducing demand elasticity. This can lead to an increase in product prices and firms' profit. Second important difference is that Adner et al. (2020) model does not capture the nuances of advertising in services, in particular, user disutility from viewing advertisements (nuisance costs). By introducing advertising in services, which impose a disutility on users, there exists a feedback loop between the two sides, i.e., advertisers and users. This generates new and distinct insights on equilibrium outcomes and welfare effects.

Platform Envelopment: The third relevant stream examines platform envelopment.[4] Previous studies have examined bundling as an entry deterrence strategy (e.g., Whinston 1990, Choi and Stefanadis 2001, Carlton and Waldman 2002, Nalebuff 2004). Some papers study bundling in platform markets with a monopoly in the primary market and non-negative price constraints (Amelio and Jullien 2012 and Choi and Jeon 2021) or exclusive content available on each platform in the complementary market (Choi 2010). Whereas, Hurkens et al. (2019) examines leveraging of market power in a setting with multi-product firms. Our analysis differs from the previous literature since we consider leveraging of market power in a distinct market structure: a market with paid hardware products and free online services with advertisements in them, and examine its implications. Using this setting, we identify conditions for the profitable leverage of market power from the product dimension to the service dimension. These conditions are specific to the platform market structure,

[4] Eisenmann et al. (2011) defines platform envelopment as an entry strategy where the entrant capitalizes on network effects in its original market to enter the network market dominated by an incumbent firm, through combined functionalities. Our work complements theirs as we explain the leverage of market power without network effects.

yielding new insights. Specifically, leveraging is more likely for small to intermediate levels of both quality advantage and nuisance cost of advertisements.

3 The Model

We consider a market with two competing firms, G and S selling differentiated hardware products, $G1$ and $S1$, differentiated online services, $G2$ and $S2$; a unit mass of users; and a unit mass of advertisers. Hardware products are paid, however, unlike previous work (Matutes and Regibeau 1988, Hahn and Kim 2012 and Kim and Hahn 2022), services are offered for free to users and purely advertising financed by placing advertisements in them (connecting advertisers to service users).

3.1 Firms

We consider a competitive setting in which both firms G and S offer a hardware product, $G1$ and $S1$, and an online service, $G2$ and $S2$. In addition, firms G and S can also decide whether to sell the product and service independently or integrate the online service with the hardware product.

Under independent pricing regime, both firms sell products and services independently. Firm $i, i = G, S$, charges a product price p_{i1} to the users. Whereas, for the service, users are charged a zero price. However, on the advertising side, firms G and S set advertising quantities, a_{G2} and a_{S2}, respectively.[5] Let r_{i2} be the price per unit of an advertisement in service, $i2, i = G, S$. Let N_{i1} be the total number of users who consume product $i1$ and N_{i2} be the total number of users who consume service $i2$, $i = G, S$. The profit of the firms are

$$\pi_G = p_{G1}N_{G1} + r_{G2}a_{G2} : \text{ Firm G's profit,} \tag{1}$$

$$\text{and } \ \pi_S = p_{S1}N_{S1} + r_{S2}a_{S2} : \text{ Firm S' profit.} \tag{2}$$

Next, consider the service integration regime when either firm G or firm S or both adopt it. For instance, if firm G adopts service integration, then its components $G1$ and $G2$ are incompatible with firm S' components $S2$ and $S1$, respectively. Therefore, users can either consume the system $G1G2$ or system $S1S2$. This implies that competition takes place between the two systems, $G1G2$ and $S1S2$.

Firm i charges a price $\tilde{p}_i, i = G, S$, for the hardware product. On the advertising side, firm i sets advertising quantity, $\tilde{a}_i, i = G, S$. Let $\tilde{r}_i$ be the price per unit of an advertisement in service, $i2, i = G, S$. Let $\tilde{N}_G$ be the total number of users who consume the system $G1G2$, and $\tilde{N}_S$ be the total number of users who consume the system $S1S2$. The profit of the firms are

$$\tilde{\pi}_G = \tilde{p}_G\tilde{N}_G + \tilde{r}_G\tilde{a}_G : \text{ Firm G's profit,} \tag{3}$$

$$\text{and } \ \tilde{\pi}_S = \tilde{p}_S\tilde{N}_S + \tilde{r}_S\tilde{a}_S : \text{ Firm S' profit.} \tag{4}$$

[5] In Appendix B.4, we have considered the alternate case with competition in per-view advertising price and show that our results remain unchanged.

3.2 Users

There is a unit mass of users who can consume at most one unit of the hardware product and the service and have a reservation value equal to zero for both. The products have different features. For example, smartphones differ in terms of size, design, camera quality, battery life, etc. Apple (product G1) is better at ease of use, and user interface, while Huawei (product S1) excels in battery life.[6] The consumers would differ in terms of their phone usability: consumers who want a very good battery life would prefer a Huawei phone, whereas those who mainly want the ease of user interface would prefer an Apple phone. Next, consider the services G2 and S2. For example, G2 is App Store and S2 is AppGallery. Both services differ in their user interface and design. As a result, each service would attract certain consumers depending on their tastes. We model this as firms competing à la Hotelling for both hardware products and services. The consumer preferences for the products and services are represented using a 1×1 unit square with firm G located at the origin (0,0) and firm S located at the coordinate (1,1). A user is characterized by a pair (x_1, x_2), where x_1 (or x_2) is her location on the Hotelling line representing her preference for the ideal product (or service). If a firm's product (or service) location does not match with her location (representing her ideal preference), then she incurs a transportation (misfit) cost from consuming the product (or service), and it is increasing in the distance between the firm's location and her location. Let t be the per-unit transportation cost for consuming the product/service. Thus, she faces a transportation cost of tx_1 (or $t(1 - x_1)$), if she consumes product G1 (or S1). Similarly, she incurs a transportation cost of tx_2 (or $t(1 - x_2)$), if she consumes service G2 (or S2). Moreover, a user obtains a standalone value V_{i1} (or V_{i2}) from consuming product $i1$ (or service $i2$), for $i = G, S$. Unlike Adner et al. (2020), we assume that users' preferences for the product and service are independent.

■ **Independent pricing.** Consider the market regime when both firms sell products and services independently. We assume that products are vertically differentiated with $V_{G1} > V_{S1}$. Using our smartphone example, iPhone's quality advantage arises from its superior integration with basic apps like alarms, calendar, etc., and access to popular third-party apps like Google Search, which is unavailable on Huawei phone. Let $\Delta = V_{G1} - V_{S1} > 0$ measure the quality advantage of firm G's product. Thus, a user's net utility from consuming product $i1$, $i = G, S$, is

$$\begin{cases} V_{G1} - p_{G1} - tx_1, & \text{if she consumes product G1, and} \\ V_{S1} - p_{S1} - t(1 - x_1), & \text{if she consumes product S1.} \end{cases} \tag{5}$$

For the service, we assume that there is no vertical differentiation with $V_{G2} = V_{S2} = W$. In addition to

[6] See "Apple iPhone 11 Pro vs Huawei P30 Pro: Which is best?," October 2019, *Stuff*, available at https://www.stuff.tv/apple-iphone-11-pro-vs-huawei-p30-pro-which-best/.

transportation costs, she incurs disutility from viewing advertisements. She is exposed to an advertising level a_{G2} (or a_{S2}), with a total disutility of δa_{G2} (or δa_{S2}), if she consumes service G2 (or S2), where $\delta \geq 0$ is the per-unit nuisance cost of advertisement. Thus, her net utility from consuming service $i2$, $i = G, S$, is

$$\begin{cases} W - \delta a_{G2} - tx_2, & \text{if she consumes service G2, and} \\ W - \delta a_{S2} - t(1 - x_2), & \text{if she consumes service S2.} \end{cases} \tag{6}$$

■ **Service Integration.** Next, consider the market regime in which either firm G or firm S or both integrate the service with the product and offers them as a system. Note that service integration by firm i would imply that its components $i1$ and $i2$ are incompatible with firm j's components $j2$ and $j1$, respectively, where $i = G, S$, and $j \neq i$. In this scenario, user choice is restricted to choose between the two systems $G1G2$ and $S1S2$.[7] A user's net utility is

$$\begin{cases} V_{G1} + W - \delta \widetilde{a}_G - \widetilde{p}_G - t(x_1 + x_2), & \text{if she consumes firm G's system } G1G2, \text{ and} \\ V_{S1} + W - \delta \widetilde{a}_S - \widetilde{p}_S - t[(1 - x_1) + (1 - x_2)], & \text{if she consumes firm S' system } S1S2. \end{cases} \tag{7}$$

3.3 Advertisers

We focus on display advertising for modeling advertising side. We assume that there is a unit mass of multi-homing advertisers who decide on whether or not to place an advertisement in firm i's service. An advertiser's expected benefit from placing an advertisement is based on three factors. One is the reach of an advertiser's product to the audience. This depends on the nature of the product. For example, baby products would attract a specific audience (parents of infants). Similarly, language learning apps would attract only users interested in learning that particular language, while messaging apps appeal to a broader demographic. We use α to define the nature of an advertiser's product. Since advertisers differ in their product characteristics, we allow α to be uniformly distributed over the interval [0,1]. As α increases, the audience appeal of the advertiser's product also increases and thus, can reach a larger audience in a firm's service. The second important factor is the baseline advertising targeting rate, i.e., how likely a user who sees the advertisement would click on it and be willing to purchase the product. For an advertiser α, let β denote the baseline advertising targeting rate, and for simplicity, we normalize it to 1. Therefore, the effective targeting rate of an advertiser α product is $1.\alpha$. The third important factor is the number of users. More users would mean that a firm can more easily find the right group of users for an advertiser. Finally,

[7] Our assumption that, under service integration, a user is restricted to choosing between the two firms' systems follows real-life practice: firms make their components incompatible with rivals' offerings. For instance, both Apple and Huawei offer the App Store and AppGallery only on their own devices, and prohibit alternative application stores on their devices. In a similar vein, Google Nest Hub and Amazon Echo Show are tightly integrated with their own voice assistants: Google Assistant and Alexa. Finally, Meta AR glasses integrate exclusively with Meta AI, while Amazon AR glasses are integrated with Alexa. In all these cases, users cannot mix components across firms and are restricted to choosing between fully integrated systems.

on the cost side, we consider a non-auction pricing mechanism in display advertisements with the selling of advertising slots based on a fixed price.

Based on this, an advertiser pays a price for a fixed number of impressions from an advertisement in service $i2$, $i = G, S$. This approach is also similar to that assumed in recent papers on online advertising (e.g., Gal-Or et al. 2018, Reisinger 2012). The net payoff of an advertiser α from advertising in service $i2$, $i = G, S$, is

$$\pi_\alpha = \begin{cases} \alpha N_{i2} - r_{i2}: & \text{Independent Pricing, and} \\ \alpha \widetilde{N}_i - \widetilde{r}_i: & \text{Service Integration.} \end{cases} \tag{8}$$

The first term of the payoff function measures the gross expected benefit to the advertiser from advertising in service $i2$, $i = G, S$. It depends on the nature of the advertiser product α (which is also the effective targeting rate), and the number of users in service $i2$, $i = G, S$, under independent pricing, i.e., N_{i2}, or service integration, i.e., $\widetilde{N}_i$. Note that given a certain number of users who join service $i2$, $i = G, S$, an advertiser with a higher α will be able to reach a larger audience, and thus, will be able to reach higher potential customers for its product, obtaining a higher expected benefit from placing an advertisement in service $i2$, $i = G, S$. The second term r_{i2} (or $\widetilde{r}_i$) is the advertising price paid under independent pricing (or service integration). In equilibrium, the advertising prices are determined so as to equate the demand for advertising slots and supply (determined by the firm i's choice of advertising quantity).

A Comment on Modelling Advertising Competition

Note that, in the baseline model, an advertiser pays an access fee (fixed price) for advertising in firm i's service, $i = G, S$. However, we have also considered different advertising models to determine the robustness of our results. In the appendix, the details are available for the following extensions: (i) competition in advertising prices instead of quantities (Appendix B.4), (ii) "pay-per-click pricing" under which a firm collects fees from advertisers every time a consumer clicks on their link (Appendix B.5), and (iii) a variant of our model to accommodate advertising via auctions (Appendix B.6). In all these extensions, our results still hold.

3.4 Timing of the Game

We solve for the subgame perfect Nash equilibrium (henceforth equilibrium) of a four-stage game:

Stage 1: Firms G and S decide whether to adopt independent pricing (N) or service integration (I). There are two distinct market regimes possible: independent pricing regime when both firms adopt independent pricing, i.e., case NN, or service integration regime when at least one firm adopts service integration, i.e., cases IN, NI, and II.

Stage 2: Firms compete in prices and advertising quantities. Under case NN, firms G and S simultaneously choose (i) user prices p_{G1} and p_{S1}, and (ii) the quantity of advertisements a_{G2} and a_{S2}. Whereas, under cases IN, NI, and II, firms G and S simultaneously choose (i) user prices $\widetilde{p}_G$ and $\widetilde{p}_S$, and (ii) the quantity of advertisements $\widetilde{a}_G$ and $\widetilde{a}_S$.

Stage 3: Advertisers decide whether to advertise in the service of firm G or S or both. Advertising prices adjust so that the demand for advertisements equals its supply.

Stage 4: Observing firms' choices, under independent pricing regime, i.e., case NN, users decide which firm's product and service to consume. Whereas, under service integration regime, i.e., cases IN, NI, and II, users decide whether to consume (i) firm G's system G1G2, or (ii) firm S' system S1S2.

4 Equilibrium Analysis

We begin by introducing two assumptions that we shall use throughout our analysis. First, we assume that a user's gross utility from consuming firm i's product, i.e., $V_{i1}, i = G, S$, and service, i.e., W are sufficiently large relative to the transportation cost parameter t to ensure that there is full market coverage under all regimes. Moreover, we focus on the scenario where both firms have positive market shares for the product, which is the interesting case in line with the anecdotal evidence.

Assumption 1. $\frac{V_{G1}+V_{S1}}{3} \geq t,\ W - \frac{3t}{2} - \frac{\delta}{2} + \delta\left[\frac{1}{4} + \frac{t^2}{\delta^2}\right]^{1/2} \geq 0,$ *and* $0 < \Delta < 3t$.

Second, we assume that users' preferences for products and services are sufficiently strong relative to the nuisance cost of advertisement. This assumption guarantees that there are gains from trade such that service integration can be a profitable strategy for a sufficiently large level of quality advantage.

Assumption 2. $t \geq 1.202 - 0.052\delta^2$.

Next, we analyze and solve for optimal prices, advertisements, demands and firms' profit for the two distinct market regimes: independent pricing (Section 4.1) and service integration (Section 4.2). Then we compare equilibrium prices and advertisements under the two regimes in Section 4.3.1, followed by the comparison of equilibrium profits in Section 4.3.2.

4.1 Independent Pricing

We characterize equilibrium under independent pricing regime, i.e., case NN, when, at *Stage* 1, both firms choose independent pricing.

■ **Stages 3 and 4**: At *Stage 4*, users make participation decisions. A type (x_1, x_2) user can make her purchase decision independently for product and service. Therefore, her choice set comprises of four options: (i) G1G2: consume product G1 and service G2, (ii) G1S2: consume product G1 and service S2, (iii) S1G2: consume product S1 and service G2, and (iv) S1S2: consume product S1 and service S2. Since x_1 and x_2 are independently and identically distributed, the demand for product and service can be analyzed separately. For the product, using Equation (5), an indifferent user is defined by the location $\hat{x}_1$ such that

$$V_{G1} - p_{G1} - t\hat{x}_1 = V_{S1} - p_{S1} - t(1-\hat{x}_1) \Rightarrow \hat{x}_1 = \frac{1}{2} + \frac{\Delta}{2t} + \frac{p_{S1} - p_{G1}}{2t},$$

where $\Delta = V_{G1} - V_{S1} > 0$ by assumption. Using this, the demand for product $i1, i = G, S$, is

$$N_{G1} = \hat{x}_1 = \frac{1}{2} + \frac{\Delta}{2t} + \frac{p_{S1} - p_{G1}}{2t}, \quad \text{and} \quad N_{S1} = 1 - \hat{x}_1 = \frac{1}{2} - \frac{\Delta}{2t} + \frac{p_{G1} - p_{S1}}{2t}. \tag{9}$$

For the service, using Equation (6), an user indifferent is defined by the location $\hat{x}_2$ such that

$$W - \delta a_{G2} - t\hat{x}_2 = W - \delta a_{S2} - t(1-\hat{x}_2) \Rightarrow \hat{x}_2 = \frac{1}{2} + \frac{\delta a_{S2} - \delta a_{G2}}{2t}.$$

Using this, the demand for service $i2, i = G, S$, is

$$N_{G2} = \hat{x}_2 = \frac{1}{2} + \frac{\delta a_{S2} - \delta a_{G2}}{2t}, \quad \text{and} \quad N_{S2} = 1 - \hat{x}_2 = \frac{1}{2} + \frac{\delta a_{G2} - \delta a_{S2}}{2t}. \tag{10}$$

At *Stage 3*, advertisers make participation decisions. Given advertising price $r_{i2}, i = G, S$, an advertiser α would advertise in service $i2$, $i = G, S$, if the marginal benefit of an advertisement is at least as large as its marginal cost, i.e., r_{i2}. Using Equation (8), the marginal advertiser $\hat{\alpha}_i$ indifferent between advertising and not advertising in service $i2$, $i = G, S$, is $\hat{\alpha}_i = \frac{r_{i2}}{N_{i2}}, i = G, S$. Using it, the level of advertisements in service $i2$, $i = G, S$, is $a_{i2} = 1 - \frac{r_{i2}}{N_{i2}}, i = G, S$. This gives the inverse advertising demand function of service $i2$, $i = G, S$, as

$$r_{i2} = (1 - a_{i2})N_{i2}. \tag{11}$$

■ **Stage 2**: Using the inverse advertising demand function defined by Equation (11), the user demand functions defined by Equations (9) and (10), and putting the values for them in the profit functions defined by Equations (1) and (2), firm $i, i = G, S$, chooses the user price p_{i1} and the advertising quantity a_{i2} to maximize its profits. Let *Stage 2* equilibrium prices, advertising quantities and demands be $p_{i1} = p^*_{i1}$, $a_{i2} = a^*_{i2}$, $N_{i1} = N^*_{i1}$ and $N_{i2} = N^*_{i2}$, $i = G, S$. The following lemma characterizes the equilibrium.

Lemma 1. *Consider the independent pricing regime, i.e., case* NN*, when both firms* G *and* S *sell products and services independently. Then Stage 2 equilibrium satisfies the following:*

(i) The equilibrium advertising quantities are characterized by

$$a^*_{G2} = a^*_{S2} = \frac{1}{2} + \frac{t}{\delta} - \sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}}. \tag{12}$$

(ii) The equilibrium prices, and demands are

$$p^*_{G1} = \frac{3t + \Delta}{3}, \; p^*_{S1} = \frac{3t - \Delta}{3}, \; \textit{and} \; N^*_{G1} = \frac{3t + \Delta}{6t}, \; N^*_{S1} = \frac{3t - \Delta}{6t}, \; N^*_{G2} = N^*_{S2} = \frac{1}{2}. \tag{13}$$

The equilibrium profit of the firms are

$$\pi^*_G = \frac{(3t + \Delta)^2}{18t} + \frac{t}{\delta}\left[\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}} - \frac{t}{\delta}\right], \; \textit{and} \; \pi^*_S = \frac{(3t - \Delta)^2}{18t} + \frac{t}{\delta}\left[\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}} - \frac{t}{\delta}\right]. \tag{14}$$

The proof of Lemma 1 is in Appendix A. Note that, for the hardware product, the equilibrium price charged by firm G (firm S) increases (decreases) with an increase in the level of firm G's quality advantage. Also, the weaker the competition (larger t), the higher the prices charged by both firms. Next, consider equilibrium advertisements. The advertising quantities depend on both the intensity of competition between firms' services and the nuisance cost of advertisements. This is because advertisements are implicit prices charged to users for accessing service $i2$, $i = G, S$. Thus, advertising competition between firms depends on the ability of users to switch (measured by per-unit transportation cost t). If users can easily switch between firms' services, then advertising competition is intense, and firms set smaller equilibrium advertising quantities to attract users. Moreover, as the nuisance cost parameter δ increases, users' aversion to advertisements increase, and thus both equilibrium advertising quantities and advertising revenue decrease.

4.2 Service Integration

We characterize equilibrium under the service integration regime, i.e., cases IN, NI, and II, when, at *Stage* 1, at least one firm chooses service integration. Under it, the competition occurs between the two systems $G1G2$ and $S1S2$.

■ **Stages 3 and 4**: At *Stage 4*, given prices $\widetilde{p}_G$ and $\widetilde{p}_S$, and advertising quantities $\widetilde{a}_G$ and $\widetilde{a}_S$, a user's choice set comprises of two options: (i) $G1G2$: consume firm G's product $G1$ and service $G2$, and (ii) $S1S2$: consume firm S' product $S1$ and service $S2$. Using Equation (7), a user indifferent between consuming system $G1G2$ and $S1S2$ is defined by a pair (x_1, x_2) such that $V_{G1} + W - \delta\widetilde{a}_G - \widetilde{p}_G - tx_1 - tx_2 = V_{S1} + W - \delta\widetilde{a}_S - \widetilde{p}_S - t(1 - x_1) - t(1 - x_2)$. Let $\widetilde{y} = \frac{x_1+x_2}{2}$ denote the average location of the indifferent user, which is given by

$$\widetilde{y} = \frac{x_1 + x_2}{2} = \frac{1}{2} + \frac{\Delta}{4t} + \frac{\widetilde{p}_S - \widetilde{p}_G}{4t} + \frac{\delta\widetilde{a}_S - \delta\widetilde{a}_G}{4t}. \tag{15}$$

The departure from the independent pricing regime, i.e., case NN, is that we have to use the density function of the average location $\widetilde{y}$ to find the demand for firm i's system, $i = G, S$. Let $\widetilde{F}(.)$ and $\widetilde{f}(.)$ denote the distribution and probability density functions of the average location $\widetilde{y}$. They are

$$\widetilde{F}(y) = \begin{cases} 2y^2, & \text{if } 0 \le y \le \frac{1}{2}, \text{ and} \\ 1 - 2(1-y)^2, & \text{if } \frac{1}{2} < y \le 1. \end{cases} \qquad \widetilde{f}(y) = \begin{cases} 4y, & \text{if } 0 \le y \le \frac{1}{2}, \text{ and} \\ 4(1-y), & \text{if } \frac{1}{2} < y \le 1. \end{cases} \tag{16}$$

An important property of the distribution of the average location (defined by Equation (16)) is that it is more peaked around the center $\left(y = \frac{1}{2}\right)$, and is less dispersed towards the extremes. This implies that with service integration, distribution of average location is less dispersed compared to the uniform distribution of locations under independent pricing. Now, using the distribution (defined by Equation (16)), the demand for each firm's system is

$$\widetilde{N}_G = \widetilde{F}(\widetilde{y}), \text{ and } \widetilde{N}_S = 1 - \widetilde{F}(\widetilde{y}). \tag{17}$$

Similar to independent pricing regime, using Equation (8), the inverse advertising demand function of service $i2, i = G, S$, is

$$\widetilde{r}_i = (1 - \widetilde{a}_i)\widetilde{N}_i. \tag{18}$$

■ **Stage 2**: Using the inverse advertising demand function defined by Equation (18), and the user demand functions defined by Equation (17), and putting the values for them in the profit functions defined by Equations (3) and (4), firm $i, i = G, S$, chooses the user price $\widetilde{p}_i$ and advertising quantity $\widetilde{a}_i$ to maximize its profits. Let *Stage 2* equilibrium prices, advertising quantities and demands be $\widetilde{p}_i = \widetilde{p}_i^*$, $\widetilde{a}_i = \widetilde{a}_i^*$, and $\widetilde{N}_i = \widetilde{N}_i^*$, $i = G, S$. The following lemma characterizes the equilibrium.

Lemma 2. *Consider the service integration regime, i.e., cases* IN, NI, *and* II, *when at least one firm chooses service integration. Then Stage 2 equilibrium satisfies the following:*

(i) The equilibrium advertising quantities are

$$\widetilde{a}_G^* = \widetilde{a}_S^* = \max\left\{\frac{1-\delta}{2}, 0\right\}. \tag{19}$$

(ii) The equilibrium prices and demands are

$$\widetilde{p}_G^* = \frac{32t^2 - (t - \Delta/2 + C)^2}{4(t - \Delta/2 + C)} - \max\left\{\frac{(1-\delta^2)}{4}, 0\right\},\ \widetilde{p}_S^* = \frac{t - \Delta/2 + C}{4} - \max\left\{\frac{(1-\delta^2)}{4}, 0\right\}, \tag{20}$$

$$\widetilde{N}_G^* = \frac{32t^2 - (t - \Delta/2 + C)^2}{32t^2},\ \textit{and}\ \widetilde{N}_S^* = \frac{(t - \Delta/2 + C)^2}{32t^2},\ \textit{with}\ C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}. \tag{21}$$

The equilibrium profit of the firms are

$$\widetilde{\pi}_G^* = \frac{[32t^2 - (t - \Delta/2 + C)^2]^2}{128t^2(t - \Delta/2 + C)},\ \textit{and}\ \widetilde{\pi}_S^* = \frac{(t - \Delta/2 + C)^3}{128t^2},\ \textit{with}\ C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}. \tag{22}$$

The proof of Lemma 2 is in Appendix A. The preceding lemma shows an interesting implication of adopting the service integration strategy. Since users are single-homing (joining only one service) and advertisers are multi-homing (advertising in both services), the firms become “bottlenecks” or “gatekeepers” providing exclusive access to the single-homing users. This gives them market power over the advertisers who wish to interact with the users and is known as a “competitive bottleneck” situation in platform markets literature (e.g., Armstrong and Wright 2007, Peitz and Valletti 2008). As a result, advertising quantities are chosen to maximize the joint surplus of the platform and each user, and explains the equilibrium advertising quantities in Lemma 2(i). For a sufficiently large nuisance cost of advertisements, i.e., $\delta \geq 1$, firms choose zero advertising levels, and it is positive otherwise. On the user side, firms compete intensively for single-homing users to attract more advertisers on the other side and extract higher advertising revenue. This implies that the firms pass on the advertising revenue to the users in the form of lower prices. As shown in Lemma 2*(ii)*, the first term captures the effect of the intensity of competition (measured by per-unit transportation cost t) on the user prices. The second term captures the subsidization of hardware products to generate more revenue on the advertising side. In fact, as Lemma 2*(ii)* shows, prices are lowered by the total amount of advertising revenue generated. Thus, in equilibrium, there is a full pass-through of advertising revenue to the users in the form of lower prices. Moreover, note that as δ increases, equilibrium advertising revenue decreases, thus prices are increasing in δ for $\delta < 1$. This implies that the pass-through is maximum at $\delta = 0$ because advertising revenues are maximized at $\delta = 0$, and vanishes for $\delta \geq 1$, because firms show no advertisements to the users. Importantly, since there is a full-pass through of advertising revenue for $\delta < 1$ and zero advertisements for $\delta \geq 1$, firms' equilibrium profit are independent of the advertising revenue, and thus nuisance cost of advertisement does not affect firms' profit.

4.3 Comparison of Pricing Regimes

4.3.1 Comparison of prices and advertising quantities

We first examine the change in equilibrium prices and advertising quantities as we move from independent pricing to a service integration regime. Comparing Lemmas 1 and 2 yields the following result.

Proposition 1. *When either firm G or firm S or both can adopt service integration, then there exist thresholds $\Delta_{pG}(\delta)$ and $\Delta_{pS}(\delta)$, where $0 < \Delta_{pG}(\delta) < \Delta_{pS}(\delta) < 3t$, such that:*

(i) *For a sufficiently large nuisance cost of advertisements, i.e., $\delta \geq 1$, service integration increases both firms' prices.*

(ii) *For a small to intermediate nuisance cost of advertisements, i.e., $\delta < 1$, service integration (a) decreases both firms' prices for a sufficiently small level of firm G's quality advantage, i.e., $0 < \Delta \leq \Delta_{pG}(\delta)$, (b) increases firm G's price and decreases firm S' price for an intermediate level of firm G's quality advantage, i.e., $\Delta_{pG}(\delta) < \Delta \leq \Delta_{pS}(\delta)$, and (c) increases both firms' prices for a sufficiently large level of firm G's quality advantage, i.e., $\Delta_{pS}(\delta) < \Delta < 3t$.*

(iii) *Service integration reduces equilibrium advertising quantities.*

Please note that we show the existence of thresholds $\Delta_{pG}(\delta)$ and $\Delta_{pS}(\delta)$ in Appendix A along with the proof of Proposition 1. The preceding proposition can be explained as follows. First, consider the equilibrium prices. The change in prices is guided by three forces: one, increased product differentiation, two, market boundary effect, and third, competitive bottleneck effect. The increased product differentiation effect is the transportation cost effect as highlighted in Kim and Hahn (2022), and results from the decreased sensitivity of users to a price change under service integration compared to the independent pricing regime. Under horizontal differentiation, note that a user must incur transportation cost when she switches to an alternative in response to a price change. Under service integration, the two products are combined with their proprietary services, and the marginal change in transportation cost increases because a user considers the change in transportation costs of both product and service in response to a change in product price. Thus, the marginal change in transportation cost is higher under service integration compared to the independent pricing regime. As a result, service integration makes users less sensitive to price changes, and product preferences become stronger. Second, as discussed in previous work (Matutes and Regibeau 1988, Hahn and Kim 2012, Hurkens et al. 2019 and Kim and Hahn 2022), service integration changes the distribution of average location of the indifferent user: more peaked around the center and thinner at the tails compared to distribution of locations under independent pricing. Therefore, the intensity of price competition depends on the average location of the indifferent consumer (or the length of the equilibrium market boundary). The closer the average location of the indifferent consumer to $\frac{1}{2}$ (or the longer the length of the equilibrium market boundary), the stronger the firm's incentive to cut prices to capture a higher demand. However, with asymmetric quality and free services, the increased product differentiation effect always dominates the effect of change in the average location of the indifferent consumer (or the length of the equilibrium market boundary) on price competition. As a result, demand becomes inelastic with service integration, thus incentivizing firms to charge higher prices.[8] Formally, to understand how pricing strategy changes, we evaluate the demand elasticities at price pair (p_{G1}^*, p_{S1}^*) and advertising levels (a_{G2}^*, a_{S2}^*) under the two regimes,[9] i.e., elasticities $\frac{p_{i1}}{N_{i1}} \cdot \frac{\partial N_{i1}}{\partial p_{i1}}$, and $\frac{\widetilde{p}_i}{\widetilde{N}_i} \cdot \frac{\partial \widetilde{N}_i}{\partial \widetilde{p}_i}$, for $i = G, S$, at (p_{G1}^*, p_{S1}^*) and (a_{G2}^*, a_{S2}^*). Under independent pricing, evaluated at (p_{G1}^*, p_{S1}^*) and (a_{G2}^*, a_{S2}^*), we know that $\frac{p_{i1}^*}{N_{i1}^*} \cdot \frac{\partial N_{i1}}{\partial p_{i1}} = -1$. Now, consider the service integration regime. Let $\widetilde{\varepsilon}_i$ denote the demand elasticity $\frac{\widetilde{p}_i}{\widetilde{N}_i} \cdot \frac{\partial \widetilde{N}_i}{\partial \widetilde{p}_i}$, evaluated at (p_{G1}^*, p_{S1}^*) and (a_{G2}^*, a_{S2}^*). It can be written as

$$\widetilde{\varepsilon}_G = -\frac{2.(3t+\Delta)(6t-\Delta)}{36t^2 - \Delta^2 + 12t\Delta}, \text{ and } \widetilde{\varepsilon}_S = -\frac{2.(3t-\Delta)(6t-\Delta)}{36t^2 + \Delta^2 - 12t\Delta}. \tag{23}$$

Note that at $\Delta = 0$, $\widetilde{\varepsilon}_G = \widetilde{\varepsilon}_S = -1$, reflecting the idea that the competition-softening effect of increased product differentiation is exactly offset by the competition-strengthening effect because average location of the indifferent user is at $\frac{1}{2}$ (or increase in the length of equilibrium market boundary). However, for $\Delta > 0$, algebraic calculations show that $-1 < \widetilde{\varepsilon}_i < 0$, $i = G, S$, because the competition-softening effect of

[8]Note that with symmetric qualities, the competition softening effect (because of reduced user sensitivity to a price change) is exactly offset by the competition strengthening effect (because average location of the indifferent user is at $\frac{1}{2}$, increasing the length of equilibrium market boundary). Thus, product prices remain unchanged with service integration. This aligns with the finding in previous work (Tabuchi 1994, Liu and Shuai 2013, Liu and Shuai 2019) that, with no quality differentiation, product prices are the same when firms are horizontally differentiated in one dimension or two dimensions. Therefore, in our model, the price differences are introduced as a result of quality asymmetry and platform market structure of services. For instance, when $\Delta > 0$, demand becomes inelastic as the competition-strengthening effect of the change in average location of the indifferent user (or the length of equilibrium market boundary) is always dominated by the price-softening effect of decreased user's sensitivity to a price change.

[9]Note that under the independent pricing, the distribution function is $F(x)$ and under service integration, it is $\widetilde{F}(x)$.

increased product differentiation dominates the effect of change in average location of the indifferent user (or change in the length of the equilibrium market boundary) on price competition. Hence, demand becomes inelastic under service integration regime. It induces firm $i, i = G, S$, to charge a product price $\widetilde{p}_i$ greater than p^*_{i1}. Finally, note that as Δ increases, the demand becomes more inelastic with service integration. Intuitively, as Δ increases, the average location of the indifferent user (defined by Equation (15)) moves closer to 1. This implies that the length of the equilibrium market boundary shortens, weakening firms' incentives to cut prices. Together with reduced user's sensitivity to a price change, demand becomes more inelastic as Δ increases.

However, as explained in the discussion of Lemma 2, service integration also gives rise to a competitive bottleneck situation. This can either result in a full pass-through of advertising revenue in the form of lower prices to users or zero advertising levels. Therefore, unlike previous studies (Matutes and Regibeau 1988, Hahn and Kim 2012 and Hurkens et al. 2019), this new effect can drive down prices with service integration. For a sufficiently large nuisance cost of advertisements, i.e., $\delta \geq 1$, from Lemma 2*(i)*, we know that firms choose zero advertising levels. As a result, there is no pass-through of advertising revenue, and firms' prices increase because of reduced demand elasticity. Whereas, for a small to intermediate nuisance cost of advertisements, i.e., $\delta < 1$, there is a full pass-through of advertising revenue in the form of lower prices and the analysis is more intricate. We find that for a small level of firm G's quality advantage, i.e., $0 < \Delta \leq \Delta_{pG}(\delta)$, the competitive bottleneck effect dominates the reduction in demand elasticity, thus both firms' prices decrease with service integration because of competing away of advertising revenue. Whereas for an intermediate level of firm G's quality advantage, i.e., $\Delta_{pG}(\delta) < \Delta \leq \Delta_{pS}(\delta)$, demand becomes sufficiently inelastic for firm G but not for firm S, leading to higher (respectively, lower) price for firm G (respectively, firm S) with service integration. For a large level of firm G's quality advantage, i.e., $\Delta_{pS}(\delta) < \Delta < 3t$, demand becomes sufficiently inelastic for both firms, which dominates the competitive bottleneck effect, thus firms' prices are higher under service integration than those under independent pricing.

Next, consider equilibrium advertising quantities. Using Equations (12) and (19), only at $\delta = 0$, the advertising quantities under independent pricing and service integration are the same and equal $\frac{1}{2}$. If $\delta \geq 1$, then service integration leads to zero advertising quantities, whereas under independent pricing, it is strictly positive. Intuitively, under service integration, users pay both a price and incur disutility from viewing advertisements, whereas, under independent pricing, platform services are free. Thus, for sufficiently high nuisance cost (disutility) of advertisements, i.e., $\delta \geq 1$, firms choose to display no advertisements under service integration. Now, consider small to intermediate nuisance cost (disutility) of advertisements, i.e., $\delta \in [0, 1)$. The comparison of advertising quantities depends on equilibrium values defined by Equations

(12) and (19). If $\frac{1-\delta}{2} < \frac{1}{2} + \frac{t}{\delta} - \sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}}$, then advertising under service integration is lower compared to the independent pricing regime. This depends on the per-unit nuisance cost of advertisement δ, and per-unit transportation cost t. While transportation cost does not affect the equilibrium advertising levels under service integration, it does affect them under independent pricing. If competition between firms is sufficiently weak (large t), then advertising under independent pricing is greater than under service integration. Under Assumption 2, this scenario holds, and service integration decreases equilibrium advertising quantities.

4.3.2 Comparison of profits

We next examine how service integration (because of at least one firm) affects the equilibrium profit of each firm.

Proposition 2. *When either firm G or firm S or both can adopt service integration, then there exist thresholds $\Delta_G(\delta)$ and $\Delta_S(\delta)$, where $0 < \Delta_G(\delta) < \Delta_S(\delta) < 3t$, such that:*

(i) For a sufficiently small level of firm G's quality advantage, i.e., $0 < \Delta \leq \Delta_G(\delta)$, both firms adopt independent pricing, i.e., case NN is an equilibrium.

(ii) For an intermediate level of firm G's quality advantage, i.e., $\Delta_G(\delta) < \Delta \leq \Delta_S(\delta)$, there is service integration because firm G adopts it, whereas firm S adopts independent pricing, i.e., case IN is an equilibrium.

(iii) For a sufficiently large level of firm G's quality advantage, i.e., $\Delta_S(\delta) < \Delta < 3t$, there is service integration because both firms adopt it, i.e., case II is an equilibrium.

(iv) As the nuisance cost of advertisements increase, the profit difference between service integration and independent pricing regimes increases for both firms, i.e., thresholds $\Delta_G(\delta)$ and $\Delta_S(\delta)$ are strictly decreasing in δ.

Please note that we show the existence of thresholds $\Delta_G(\delta)$ and $\Delta_S(\delta)$ in Appendix A along with the proof of Proposition 2. Figure 1 illustrates Proposition 2. In Figure 1, the thresholds $\Delta_G(\delta)$ and $\Delta_S(\delta)$ represent the loci of points along which firm G and firm S, respectively obtain same profit under independent pricing and service integration regimes. To understand the intuition behind the preceding proposition, we decompose the effect of service integration (by at least one firm) on profitability into three different effects: market segmentation effect, demand elasticity effect, and competitive bottleneck effect. We discuss each of the following effects in detail:

(i) The market segmentation effect arises because service integration changes each firm's market share for both product and service. First, consider the demand for firm G's system $G1G2$. Service integration increases the demand for the service $G2$ and reduces the demand for the product $G1$. Intuitively, firm G leverages its quality advantage to increase demand for its service G2. However, compared to the independent pricing regime, this reduces its demand for product G1 because of a higher mismatch between consumer preferences and hardware G1 location. As a result, demand for firm S' product S1 increases. To see this, consider the demand for the system G1G2 at price pair (p^*_{G1}, p^*_{S1}). Using the average location of the indifferent user (defined by Equation (15)) and the distribution function (defined by Equation (16)), it is $\frac{1}{2} + \frac{\Delta}{6t}\left(1 - \frac{\Delta}{12t}\right)$. Since, for $\Delta > 0$, it is strictly greater than $\frac{1}{2}$, the demand for service G2 increases with integration. Note that, as Δ increases, more users purchase service G2 because system G1G2 becomes more attractive. Moreover, at price pair (p^*_{G1}, p^*_{S1}), the demand for product G1 is $\frac{1}{2} + \frac{\Delta}{6t}\left(1 - \frac{\Delta}{12t}\right) < \frac{3t+\Delta}{6t}$. Next, consider the demand for firm S' system S1S2. Evaluated at price pair (p^*_{G1}, p^*_{S1}), it is $\frac{1}{2} - \frac{\Delta}{6t}\left(1 - \frac{\Delta}{12t}\right)$. For all $\Delta > 0$, it is less than $\frac{1}{2}$, but greater than $\frac{3t-\Delta}{6t}$. Thus, the number of users joining firm S' service S2 decreases but more users are now purchasing its product S1. Note that, as Δ increases, then with service integration, the demand increase for product S1 is higher because the number of users who switch to consume system S1S2 is higher.

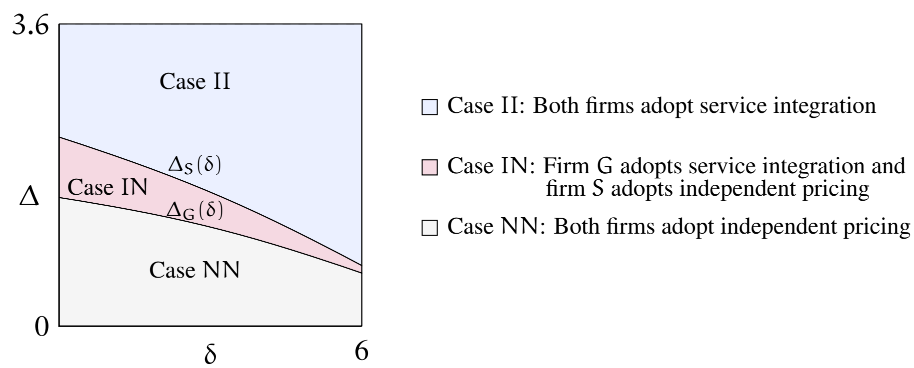


Figure 1: Comparison of profits under different market regimes ($t = 1.21$).

(ii) As discussed earlier, the demand elasticity effect stems from two distinct mechanisms. First, service integration decreases users' sensitivity to a price change. A user must incur higher transportation costs when she switches to an alternative in response to a price change. This increases differentiation between the two hardware devices relative to the independent pricing regime (where consumers can use either service with hardware). Hence, consumer preferences become stronger and are less likely to switch in response to a price increase. Second, service integration also affects the average location of the indifferent user (or the length of the equilibrium market boundary). Prior literature (like Matutes and Regibeau 1988, Hahn and Kim 2012) showed that if the average location of the indifferent user is close to $\frac{1}{2}$ (or the length of the equilibrium market boundary increases), price competition intensifies because firms have stronger incentives to cut prices to capture more demand. However, unlike previous work, in our model with asymmetric quality and free services, the effect of reduced user sensitivity to a price change always dominates the market boundary effect for any positive quality advantage, i.e., $\Delta > 0$. As a result, demand becomes inelastic with service integration compared to independent pricing. Thus, it induces each firm to charge a higher price. Moreover, as discussed in Section 4.3.1, under service integration, demand becomes more inelastic as Δ increases because the average location of the indifferent user moves closer to 1 (or the length of the equilibrium market boundary decreases).

(iii) Finally, the competitive bottleneck effect arises because with service integration, firm $i, i = G, S$, provides multi-homing advertisers, an exclusive access to single-homing users. As discussed earlier, for small δ, a firm cannot appropriate the advertising gains and there is a full pass-through of advertising revenue to users in the form of lower prices. Whereas, for large δ, there is no pass-through effect as firms choose zero advertisements with service integration. As a result, the competitive bottleneck effect results in firms' profit being independent of advertising revenue with service integration (refer Lemma 2), whereas they earn positive advertising revenue under independent pricing, which is decreasing in δ (refer Lemma 1). Therefore, the negative profit impact of competitive bottleneck effect decreases with an increase in δ.

The profit comparison depends on the strength of the three effects, which are determined by the market characteristics: per-unit nuisance cost δ and the level of firm G's quality advantage Δ. First, consider the case when δ is small to intermediate, i.e., $\delta < 1$. In this case, competitive bottleneck effect leads to a full pass-through of advertising revenue in the form of lower prices to users. For a small level of quality advantage, i.e., $0 < \Delta \leq \Delta_G(\delta)$, the market segmentation effect is weak for both firms. This occurs because the average location of the indifferent user is close to $\frac{1}{2}$. For firm G, the increment in demand for its service S2 is small. Similarly, firm S' increment in market share for the product S1 is small. Moreover, demand elasticity effect is weak, implying that a possible price increase is small. Thus, the increment in market shares for the two firms and decreased demand elasticity is dominated by the competitive bottleneck effect, reducing each firm's profit. Next, consider a sufficiently large level of quality advantage, i.e., $\Delta_S(\delta) < \Delta < 3t$. Firms' products become strongly differentiated with service integration and dominate the competitive bottleneck effect, softening price competition between firms. Moreover, the market segmentation effect is strong for both firms because firm G gains sufficient users for its service G2, and increase in demand for firm S' product S1 is also sufficiently large. Together, strong market segmentation and higher prices dominate the inability to appropriate advertising revenue, thus both firms benefit from service integration. However, for an intermediate level of quality advantage, i.e., $\Delta_G(\delta) < \Delta < \Delta_S(\delta)$, competitive bottleneck effect has an asymmetric effect on firms' profit. For firm G, demand becomes sufficiently inelastic and together with an increase in demand for service G2, dominate the competitive bottleneck effect, thus increasing its profit.

Whereas, for firm S, competitive bottleneck effect dominates the decrease in demand elasticity and increase in product $S1$ demand, thus decreasing its profit.

Next, when δ is large, i.e., $\delta \geq 1$, the competitive bottleneck effect results in firms displaying no advertisements in services. Thus, prices always increase (because of reduced demand elasticity), and advertising revenue decreases with service integration. When quality advantage is sufficiently large, for firm i, $i = G, S$, service integration increases price sufficiently, and market segmentation effect is also strong. Together, they dominate the decrease in advertising revenue, thus increasing the profit of firm i, $i = G, S$; otherwise its profit decreases.

Finally, consider part *(iv)* of Proposition 2. Under independent pricing, as the per-unit nuisance cost of advertisement increases, firms reduce their advertising levels to avoid losing users, which decreases their advertising revenue. Whereas under service integration, as described above, competitive bottleneck effect results in firms' profit being independent of the the advertising revenue. As a result, for a given level of quality advantage Δ, an increase in the per-unit nuisance cost of advertisement does not affect firms' profit under service integration but reduces advertising revenue obtained under independent pricing. Therefore, as δ increases, a smaller level of Δ is required to make demands sufficiently inelastic for firms to offset the competitive bottleneck effect with service integration. Thus, the thresholds $\Delta_G(\delta)$ and $\Delta_S(\delta)$ decrease with an increase in δ.

We briefly discuss two important implications of Proposition 2. For a small to intermediate quality advantage and sufficiently large nuisance cost of advertisements, service integration can increase both firms' profit. This is because the negative profit impact of competitive bottleneck effect is weak and dominated by the reduced demand elasticity and strong market segmentation effects. Moreover, for a small to intermediate levels of both quality advantage and nuisance cost of advertisements, there is a potential for anti-competitive effect, specific to platform markets. For instance, when $\delta < 1$, the two-sided market structure of the services provides a new justification for firm G to credibly leverage its quality advantage (dominance) to the service dimension. With service integration, the competitive bottleneck situation has an asymmetric effect on firms' profit. For firm G, the quality advantage makes its demand sufficiently inelastic and together with increased market share for service G2, dominates the competitive bottleneck effect, thus increasing its profit. However, firm S' profit decreases because competitive bottleneck effect dominates the reduced demand elasticity and higher demand for product $S1$. This insight about the rationale for leverage of market power in the presence of advertising financed platforms cannot be inferred from previous literature (e.g., Hahn and Kim 2012, Hurkens et al. 2019, Adner et al. 2020).

5 Extensions

In this section, we discuss a few extensions to the baseline model.

5.1 Free Online Services with No Advertisements

In this extension, we look at a scenario when firms' services do not have advertisements in them. In the absence of an advertising side, a user's nuisance cost of advertisements from consuming either service G2 or S2 equals zero under both market regimes. Moreover, firm i's profit is $p_{i1}N_{i1}$ under independent pricing, and $\widetilde{p}_i\widetilde{N}_i$ under service integration, where $i = G, S$. Relegating the technical details to Appendix B.1, our main result is summarized in the following proposition.

Proposition 3. *Consider competition between firms G and S offering services with no advertisements in them. Then, service integration always increases both firms' prices and profit.*

The proof of Proposition 3 is in Appendix B.1. Similar to the analysis in the baseline model, the profitability of service integration depends on how it changes firm i's market share and the price elasticity of demand. Under independent pricing, firms' profit from product sales do not depend on the intensity of competition in the service dimension. However, with service integration, firms compete in prices, taking into account the demand for the entire system. As discussed earlier, there is an increase in product differentiation because of an increase in the strength of users' preferences for the two products. Moreover, for $\Delta > 0$, the increased product differentiation effect outweighs the change in average location of the indifferent consumer with service integration and thus makes demand inelastic for all $\Delta > 0$. Finally, with no advertisements, the competitive bottleneck effect vanishes, and firms do not have the incentive to lower prices to attract more users for advertisers. Thus, unlike Matutes and Regibeau (1988), the reduced demand elasticity always leads to higher prices and profit under service integration for all levels of quality advantage $\Delta > 0$.

5.2 Asymmetric Intensity of Competition

Our baseline model assumes that per-unit transportation cost for product and service is the same and equals t. The current extension examines the case when users' preferences are weaker for service than for product, captured by lower per-unit transportation cost for service than for product. Let t_1 and t_2 denote the per-unit transportation cost of consuming the product and service, respectively, with $t_2 < t_1 = t$. Under independent pricing, a user obtains net utility $V_{G1} - p_{G1} - t_1x_1$, if she consumes product G1, and $V_{S1} - p_{S1} - t_1(1-x_1)$, if she consumes product S1. Moreover, a user obtains net utility $W - \delta a_{G2} - t_2x_2$, if she consumes service G2, and $W - \delta a_{S2} - t_2(1-x_2)$, if she consumes service S2. Under service integration, a user with location (x_1, x_2) obtains net utility $V_{G1} + W - \delta\widetilde{a}_G - \widetilde{p}_G - (t_1x_1 + t_2x_2)$, if she consumes system G1G2, and $V_{S1} + W - \delta\widetilde{a}_S - \widetilde{p}_S - (t_1(1-x_1) + t_2(1-x_2))$, if she consumes system S1S2. The details of equilibrium outcomes are relegated to Appendix B.2. Due to the complexity of analysis, we conduct numerical analysis with $t_1 = 1.2$ and $t_2 = 0.9$ to bring out the main insights (illustrated in Figure 2). As shown in Figure 2, our main results hold for sufficiently large δ. However, in contrast to the baseline model, for small δ, service integration always decreases firm S' profit. Intuitively, for $t_2 < t_1 = t$, like in the baseline model (with $t_2 = t_1 = t$), when a user switches to an alternative in response to a price increase, the marginal change in transportation cost is greater under service integration than under independent pricing. However, in contrast to the baseline model, the marginal increase in transportation cost is weaker. This implies that, compared to the service integration regime under the baseline model, the reduction in demand elasticity is weaker.

For sufficiently small δ, firms obtain large advertising revenue under independent pricing. If there is service integration, then for firm S, the demand elasticity effect is dominated by the competitive bottleneck effect, reducing its profit for all $\Delta > 0$. However, for firm G, the reduced demand elasticity and the increase in demand for service dominate the competitive bottleneck effect for sufficiently large Δ, thus increasing its profit and decreasing it otherwise. For sufficiently large δ, advertising revenue is small under independent pricing, and the intuition for the profit impact of service integration remains the same as under the baseline model.

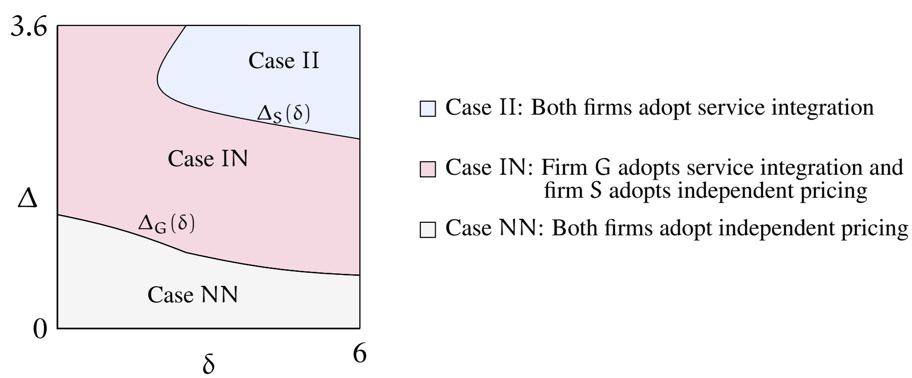


Figure 2: Comparison of profits under asymmetric intensity of competition ($t_1 = 1.21$ and $t_2 = 0.9$).

5.3 Multi-Homing Users

We now consider the possibility that users can multi-home, and some users can consume both services. Under independent pricing, a user's utility from consuming either product G1 or S1 is defined by Equation (5). If a user single-homes and consumes either G2 or S2, then her utility is given by Equation (6). If a user multi-homes and consumes both services, then her utility is $2W - \delta a_{G2} - \delta a_{G2} - t$. The user indifferent between single-homing on service G2 and multi-homing is given by $x_2^m = \frac{W-\delta a_{G2}}{t}$, giving demand for service G2, $N_{G2} = x_2^m = \frac{W-\delta a_{G2}}{t}$. Similarly, the user indifferent between single-homing on service S2 and multi-homing is given by $x_1^m = 1 - \left[\frac{W-\delta a_{S2}}{t}\right]$, giving demand for service S2, $N_{S2} = 1 - x_1^m = \frac{W-\delta a_{S2}}{t}$. Note that, under independent pricing, two firms are not in competition with each other. The absence of a rival's strategic response to firm i reduction in ads leads to a greater incentive to reduce ads. This, in turn, results in higher demand for services under user multi-homing compared to the baseline case when all users are single-homing. As a result, under independent pricing, each firm obtains higher demand for services, which generates higher advertising revenue from services compared to the baseline framework with all users single-homing. Next, under service integration, a user cannot multi-home, because a firm's product and service become incompatible with the rival system, and the equilibrium outcome remains the same as defined in Lemma 2. Relegating the technical details to Appendix B.3, the following remark describes our numerical results when users can choose one or both services and is illustrated in Figure 3.

Remark 1. *When users are allowed to choose one or both services, then the following holds:*

(i) For small to intermediate nuisance cost of advertisements, in contrast to the baseline model, service integration always decreases firm S' profit, whereas, firm G's profit increases for sufficiently large quality advantage, and decreases otherwise.

(ii) For a sufficiently large nuisance cost of advertisements, our results coincide with the baseline model. Service integration improves both firms' profit for a sufficiently large quality advantage, and decreases both firms' profit for sufficiently small quality advantage. Whereas, for an intermediate quality advantage, it increases firm G's profit, and decreases firm S' profit.

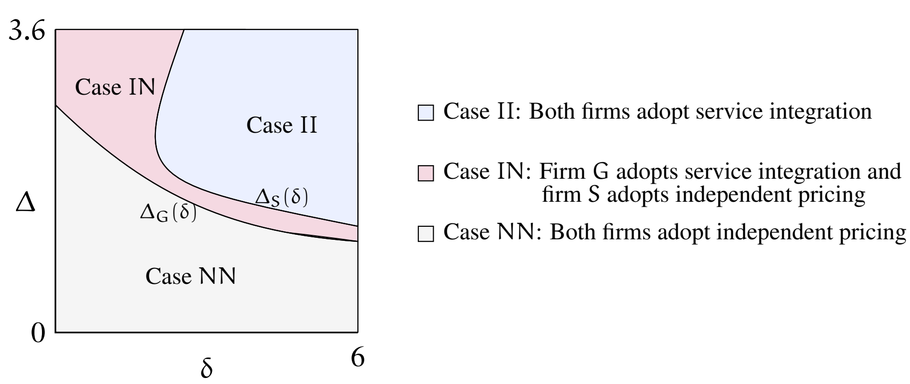


Figure 3: Comparison of profits when users can choose one or both services ($t = 1.21$ and $W = 1.5$).

Consider small to intermediate nuisance cost of advertisements. Under independent pricing, demand for two services is large, thus advertising revenue obtained is large. With service integration, for firm S, the decrease in advertising revenue (because of the competitive bottleneck effect) dominates the reduced demand elasticity, thus decreasing its profit for all $\Delta > 0$. However, for firm G, when the quality advantage is sufficiently large, the demand becomes sufficiently inelastic, which dominates the decrease in advertising revenue (because of the competitive bottleneck effect), thus increasing its profit with service integration.

5.4 Single-Homing Advertisers

In this section, we consider an alternate setting with single-homing advertisers, and informally comment on how results might change. Since there is competition on the advertising side, in equilibrium, an advertiser who chooses to place an advertisement in the service platform must be indifferent between service G2 and S2. Under independent pricing, suppose the marginal advertiser is denoted by $\bar{\alpha}$. This implies that an advertiser α, where $\alpha \in [\bar{\alpha}, 1]$, must satisfy the condition, $\alpha N_{G2} - r_{G2} = \alpha N_{S2} - r_{S2} \geq 0$, and $\bar{\alpha} N_{G2} - r_{G2} = \bar{\alpha} N_{S2} - r_{S2} = 0$. Solving for the equilibrium advertising levels yields $a^*_{G2} = a^*_{S2} = \frac{\delta+3t-\sqrt{\delta^2+9t^2-2\delta t}}{4\delta}$. We first compare it with advertising levels in the baseline model. First, unlike the baseline model, since advertisers single-home, there is intensified advertising competition. This puts downward pressure on equilibrium advertisements. Second, advertisements are less responsive to an increase in per-unit nuisance cost δ when advertisers single-home compared to the baseline case when advertisers are allowed to multi-home. This can be explained as follows. Note that like in the baseline model, as δ increases, users see advertisements more annoying, which intensifies competition for users, and puts downward pressure on advertising levels (to attract more users). However, in addition, there is also competition on the advertising side. So, a lower advertising level would incentivize rival to decrease advertising level, which would further intensify user side competition. This is the competition dampening effect of δ, not present in the baseline model, and counters the effect of increased nuisance cost on users. Therefore, advertisements are less responsive to an increase in per-unit nuisance cost δ, when advertisers single-home compared to the baseline case when advertisers are allowed to multi-home. Moreover, the competition dampening effect is stronger for higher levels of δ. As a result, for small levels of δ, the competition dampening effect is weak and outweighed by the intensified advertising competition effect. This leads to lower advertising levels and revenue when advertisers single-home compared to the baseline case when advertisers are allowed to multi-home. Conversely, for large δ, the competition dampening effect is strong and dominates the intensified advertising competition effect, leading to higher advertising levels and revenue when advertisers single-home compared to the baseline case when advertisers are allowed to multi-home.

Under service integration, since users are single-homing (joining only one service), the firms still provide exclusive access to the single-homing users. As a result, like in the baseline model, we have a "competitive bottleneck" situation, and the advertising quantities are chosen to maximize the joint surplus of the platform and each user (as defined in Lemma 2*(i)*). This in turn implies that firms' equilibrium profits are independent of the advertising revenues, and thus, nuisance cost of advertisement does not affect firms' profit.

Since profits are independent of advertising revenue under service integration and decreases in nuisance costs under independent pricing, our results on profit comparison will carry over to this extension. However, in contrast to the baseline case, for small levels of δ, advertising revenue is lower under independent pricing. Therefore, service integration is more likely for small levels of δ. Whereas, for large levels of δ, advertising revenue is higher under independent pricing compared to the baseline case. Therefore, service integration is less likely for large levels of δ.

6 Contributions

We briefly compare our results to the most closely related papers by Matutes and Regibeau (1988), Hahn and Kim (2012), Hurkens et al. (2019), and Adner et al. (2020).

6.1 Product incompatibility with symmetric qualities and horizontal differentiation

In a two-dimensional Hotelling model with two firms, each owning two paid components, Matutes and Regibeau (1988) show that incompatibility is not profitable because it intensifies price competition through internalization of price cuts. Kim and Hahn (2022) clarified the mechanisms in Matutes and Regibeau (1988) by explaining two effects of incompatibility. The first effect is due to change in the length of the equilibrium market boundary. A longer equilibrium market boundary results in intensified price competition. Second effect results from increased transportation cost because when a user switches to an alternative in response to a price change, the marginal change in transportation cost is higher (leading to competition softening). Using this reasoning, in Matutes and Regibeau (1988), incompatibility increases the length of equilibrium market boundary, which dominates the increased transportation cost effect, thus intensifying price competition and decreasing firms' profit. However, unlike Matutes and Regibeau (1988), in our model, service integration always reduces firms' demand elasticities because transportation cost effect dominates the change in the length of the equilibrium market boundary for all $\Delta > 0$. Next, we clearly delineate how our model extends the work of Matutes and Regibeau (1988) by first presenting a benchmark case and then introducing additional features (quality asymmetry and advertisements) one by one.

Benchmark case: symmetric qualities without advertising: We begin with a benchmark case with $\Delta = 0$ and no advertisements. Using Lemmas 1 and 2, we can obtain $p_i^* = \widetilde{p}_i^* = t$, $i = G, S$ and $\pi_i^* = \widetilde{\pi}_i^* = t/2$, $i = G, S$. Intuitively, when $\Delta = 0$, the length of equilibrium market boundary increases. However it is exactly offset by the transportation cost effect, and firms' demand elasticities remain unchanged with service integration. This can be formally determined by setting $\Delta = 0$ in Equation (23), which shows that firms' demand elasticities remain unchanged with service integration and equal - 1. Since there are no ads, competitive bottleneck vanishes, thus firms do not have price reduction incentives. Therefore, equilibrium prices and profits remain unchanged with service integration.

Introducing quality asymmetry: With quality asymmetry, i.e., $\Delta > 0$, and no advertisements in service, we find that, unlike Matutes and Regibeau (1988), service integration increases firms' prices and profit (Proposition 3). As discussed earlier, for $\Delta > 0$, service integration makes firms' demand inelastic (refer Equation (23)) because the transportation cost effect dominates the change in the length of the equilibrium market boundary for all $\Delta > 0$. Moreover, with no advertisements, and the absence of competitive bottleneck effect, firms do not have to pass through advertising revenue in the form of lower prices. Thus, the reduced demand elasticity leads to higher prices and profit under service integration for all $\Delta > 0$.

Introducing advertisements in services: Unlike Matutes and Regibeau (1988), in our asymmetric framework, the price and profit reduction with service integration can result only when free services include ads and users dislike advertisements. Under this platform market structure of the services, service integration can lead to a full pass-through of advertising revenue to users in the form of lower prices. The second component, i.e., $\max\left\{\frac{1-\delta^2}{4}, 0\right\}$, in $\widetilde{p}_i^*, i = G, S$, (defined in Lemma 2) captures the possibility of intensified price competition due to the specific nature of the platform market, i.e., lowering prices to attract users for the advertisers. Since there is a full pass-through of advertising revenue (for $\delta < 1$) or firms choose zero advertisements (for $\delta \geq 1$), firms' profit are independent of advertising revenue under service integration, whereas they earn positive advertising revenue under independent pricing. As a result, distinctly from the Matutes and Regibeau (1988), the asymmetric qualities and the platform structure of services in our framework explain the change in profit with service integration which depends on the level of quality advantage and nuisance cost of advertisements. Importantly, Matutes and Regibeau (1988) result is completely reversed either for sufficiently large quality advantage or small to intermediate quality advantage with large nuisance cost of advertisements. For this parameter range, service integration reduces demand elasticity sufficiently which dominates the negative effect of competitive bottleneck situation, and thus increases both firms' profit.

6.2 Product incompatibility with asymmetric qualities and horizontal differentiation

Previous work has examined the competitive effects of pure bundling in a two-dimensional Hotelling framework with asymmetric rivals, where one firm holds a dominance across markets due to either a cost advantage (Hahn and Kim, 2012) or a quality advantage (Hurkens et al., 2019). These studies show that incompatibility changes the distribution of consumers' average locations—making it more peaked at the center and thinner at the tails—relative to the distribution of locations under compatibility. As a result, the effect on firms' profit depends on how the average location of the indifferent consumer (or the length of the equilibrium market boundary) changes under incompatibility, and how this impacts price competition. When dominance is small (due to a low cost or quality advantage), the average location of the indifferent user is located near the center of the distribution. This leads to intensified price competition due to a longer equilibrium market boundary under incompatibility, thereby reducing profits. In contrast, when dominance is large, the average indifferent consumer shifts toward the tail of the distribution, which weakens price competition and increases firms' profit. We next compare our findings with Hahn and Kim (2012) and Hurkens et al. (2019) by first introducing only quality asymmetry, and then including advertisements in services.

Introducing quality asymmetry: We first introduce only quality asymmetry, i.e., $\Delta > 0$, while still omitting advertising. In contrast to Hahn and Kim (2012) and Hurkens et al. (2019), we show that service integration always makes demand inelastic, irrespective of the average location of the indifferent consumer. This is because the increased transportation cost of switching to a rival system always dominates the change in the average location of the indifferent consumer, leading to inelastic demand with service integration for all $\Delta > 0$. To highlight this, if there are no advertisements, and $\Delta > 0$, then Proposition 3 shows that both prices and profit increase with service integration, because of the inelastic demand for both firms.

Introducing advertisements in services: Next, when we introduce advertising in the services, then unlike Hahn and Kim (2012) and Hurkens et al. (2019), we introduce a new effect of incompatibility based on the interaction between users and advertisers under the platform market structure of services, which can reverse the results in previous studies. For instance, for small to intermediate quality advantage and sufficiently large nuisance cost of advertisements, in contrast to Hahn and Kim (2012) and Hurkens et al. (2019), we find that service integration increases both firms' profit. Since the advertising revenue obtained under independent pricing is small, the reduction in profit because of competitive bottleneck situation with service integration is small and dominated by the reduced demand elasticity (leading to higher prices).

Moreover, in the presence of advertisements in services, our results also identify novel conditions for firm G to credibly leverage its quality advantage (dominance) to the service market. As discussed in Section 4.3.2, for an intermediate quality advantage and $\delta < 1$, service integration (because of firm G) results in a full pass-through of advertising revenue to users through lower prices, which affects firms asymmetrically: increasing firm G's profit and decreasing firm S' profit. Therefore, the platform market structure of the services provides a new rationale for leverage of market power which cannot be inferred from Hahn and Kim (2012) and Hurkens et al. (2019).

6.3 Platform compatibility

When all consumers have independent preferences for product and software, Adner et al. (2020) find that incompatibility always decreases firms' prices and profit. This crucially depends on the stronger preferences for product relative to software, which implies that a user's sensitivity to a price change remains unchanged with incompatibility. In contrast, in our paper, a user's sensitivity to a price change decreases with service integration, outweighing competition strengthening effect of change in average location of the indifferent user, thus demand becomes inelastic. Hence, with independent consumer preferences, unlike Adner et al. (2020), we find that incompatibility can increase firms' prices and profit for sufficiently large Δ or small to intermediate Δ and large δ. Next, with correlated preferences, Adner et al. (2020) find that asymmetric incompatibility can improve firms' profits as the superior firm relies on higher hardware revenue, and the inferior hardware firm relies on higher content sales revenue to profit from incompatibility. Unlike Adner et al. (2020), in our paper, the profitable components for the two firms are distinct, i.e., the superior hardware firm G relies on higher service sales, and the inferior hardware firm S relies on higher hardware sales to profit from service integration. This crucially hinges on the modelling of nuisance cost of advertisements, because of which service integration reduces equilibrium advertising quantities. This implies that effective monetization of services would require a large user base. Since firm G has the quality advantage, it can leverage it to increase its service user base significantly and attain higher revenue. However, firm S, being an inferior product firm, cannot monetize its service effectively. Thus, it can profit from service integration only if its demand becomes sufficiently inelastic to drive up its hardware revenue. Moreover, as discussed above, the two-sided structure of services generates novel insights regarding the leverage of market power, which cannot be inferred from Adner et al. (2020).

7 Welfare Analysis

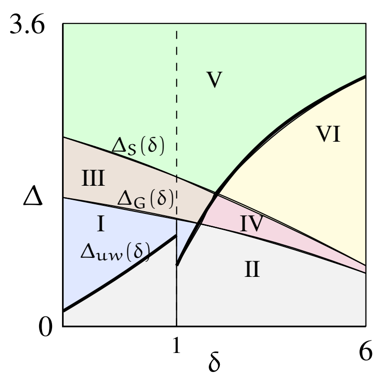
We now examine the impact of service integration on user welfare and advertisers' profit. In order to understand the welfare effects of service integration, we need to look at the various trade-offs involved. First, service integration increases users' gross surplus as more users will be consuming the system $\mathsf{G1G2}$ which has a larger standalone value. Second, it increases total transportation costs because of the redistribution of users across the two firms. Now, users are restricted from choosing the most preferred system, and thus, there is a larger mismatch between their preference and the system location. Third, from Proposition 1*(i)* and *(ii)*, we know that the effect of service integration on prices can be non-monotonic and depends on the strength of the reduced demand elasticity and competitive bottleneck effects. Fourth, Proposition 1*(iii)* implies that service integration decreases total nuisance cost of advertisements because of lower advertisements. The following proposition summarizes the main result on welfare analysis.

Proposition 4. *When either firm G or firm S or both can adopt service integration, then there exists a threshold $\Delta_{uw}(\delta)$, such that:*

(i) For a small level of firm G's quality advantage, i.e., $0 < \Delta \leq \Delta_{uw}(\delta)$ service integration increases user welfare and decreases it otherwise.

(ii) Service integration decreases advertisers' profit.

Please note that we show the existence of threshold $\Delta_{uw}(\delta)$ in Appendix A along with the proof of Proposition 4. The intuition for the preceding proposition is as follows. First, consider user welfare. For small level of quality advantage, i.e., $0 < \Delta \leq \Delta_{uw}(\delta)$, increase in users' gross surplus and savings in total nuisance cost (because of reduced advertisements) dominate the change in prices paid for the two systems, and increased total transportation cost, thus increasing user welfare. The reverse holds for a sufficiently large quality advantage, i.e., $\Delta_{uw}(\delta) < \Delta < 3t$. Demand becomes sufficiently inelastic, raising the price paid for the system and total transportation cost increases. Together, these two effects dominate the decrease in total nuisance cost (because of reduced advertisements) and increase in users' gross surplus, thus decreasing user welfare. Next, service integration also decreases advertisers' profit because of fewer advertisements.



Region I: Both firms' profit, and user welfare fall and advertisers' profit fall with service integration
Region II: Both firms' profit fall and user welfare rises and advertisers' profit fall with service integration
Region III: Firm G's (firm S') profit rises (falls), user welfare falls and advertisers' profit fall with service integration
Region IV: Firm G's (firm S') profit rise (falls), user welfare rises and advertisers' profit fall with service integration
Region V: Both firms' profit rises, user welfare falls and advertisers' profit fall with service integration
Region VI: Both firms' profit rises, user welfare rises and advertisers' profit fall with service integration

Figure 4: Comparison of profits, user welfare and advertisers' profit under different market regimes ($t = 1.21$).

Figure 4 illustrates the preceding discussion. As can be seen from the figure, there is a threshold $\Delta_{uw}(\delta)$, which represents the loci of points along which user welfare is the same under independent pricing and service integration. Moreover, Figure 4 also highlights six subregions differentiated based on how service integration affects firms' profit, user welfare, and advertisers' profit. In Regions I, II, V, and VI, service integration is either not profitable (Regions I and II) or profitable for both firms (Regions V and VI). However, it can have asymmetric welfare effects in Regions II and VI, i.e., increasing user welfare and decreasing advertisers' profit; otherwise, both users and advertisers suffer (Regions I and V). From a policy point of view, Regions III, IV, and VI are important. In these three regions, service integration either has an asymmetric effect on firms' profit (Regions III and IV) or increases both firms' profit (Region VI). However, the welfare implications of a profitable service integration vary as user welfare decreases (Region III) or increases (Regions IV and VI), and advertisers' profit always decreases. In Regions III and IV, an anti-competitive service integration by firm G reduces firm S' profit, however, it has different policy implications depending on the level of nuisance cost of advertisements.

8 Discussion and Conclusion

Digital markets have seen the rise of dominant firms adopting "walled garden" approach through service integration, where firms make their products and services incompatible with rival systems. We examine firms' incentives to adopt service integration when each firm offers a paid hardware product and a free, advertising-financed service. Our analysis reveals several key findings. Service integration has a non-monotonic effect on prices and decreases advertising levels. When dominant firm G's quality advantage is large, the reduced demand elasticity effect outweighs competing away of advertising revenues through lower prices, thus increasing profit of both firms. The reverse holds when the dominant firm G's quality advantage is sufficiently small. For small to intermediate levels of both quality advantage and nuisance cost of advertisements, service integration benefits the dominant firm G but harms rival firm S, whereas for sufficiently large nuisance cost of advertisements, service integration benefits both firms. Finally, from a welfare perspective, a profitable service integration can increase user welfare but decreases advertisers' profit.

8.1 Managerial Implications

Our findings offer insights for platform owners on how to manage and design their products and services.

8.1.1 Implications for Compatibility Incentives

Our results show that when the difference in product qualities is sufficiently large, then both firms benefit from service integration. In the smartphone market, Apple phone provides better functionalities due to its efficient integration with other Apple devices and applications, and offers a large number of popular apps, relative to the Huawei phone. As a result, there is a significant difference in the functionalities of the two smartphones, giving Apple a quality advantage. Consistent with our model results, both smartphones would be better off keeping their systems incompatible with each other. A tighter integration between Apple and App Store helps Apple to strategically expand its App Store's user base for monetizing through advertisements in it. Similarly, keeping Huawei phone and AppGallery better integrated helps it to focus on improving its device sales through improving its smartphone user base. Likewise, in the smart displays market, Google Nest Hub provides more features/functionalities due to its integration with the Google ecosystem, relative to Amazon Echo Show, giving Google Nest Hub a quality advantage. Embracing service integration enables Google to strategically focus on expanding Google Assistant's user base and monetizing them through advertisements. Similarly, keeping Echo Show and Alexa integrated helps Amazon to improve profits by expanding its Echo Show user base. Finally, in the smart glass market, keeping smart glass tightly integrated with its own proprietary assistant, enables Meta and Amazon to focus on different revenue components. Meta (with a dominant market share) can find a new opportunity for ad-based monetization while Amazon can focus on smart glass sales.[10]

8.1.2 When is Leveraging of Market Power through Service Integration Strategy Profitable?

Another interesting implication of our result is identifying the market conditions under which a firm with a superior product in digital markets can leverage its market power to the service dimension. The two-sided market structure of the services results in a new rationale for the dominant firm to credibly leverage its quality advantage (dominance) to the service dimension. Specifically, for small to intermediate levels of both quality advantage and nuisance cost of advertisements, service integration leads to competing away of advertising revenue in the form of lower prices, which can have asymmetric effects on firms' profit. It increases dominant firm's profit, whereas inferior quality firm's profit decreases.

8.1.3 Implications for Optimal Design of the Online Services

Our results also suggest a number of strategies that a rival online service can consider in the face of the threat of leveraging market power by a dominant firm. This threat arises when a dominant firm with superior product quality adopts service integration to extend its market power from hardware to services, harming the rival firm through both reduced prices and advertising revenue. Therefore, an important strategic choice is to invest and match the product quality/ functionalities of the dominant firm. In this case, based on our results, leverage of market power does not arise. Moreover, in our model, for small to intermediate levels of quality advantage, the likelihood of leveraging is affected by the level of per-unit nuisance cost of advertisement (refer Figure 1). This highlights the role of incorporating user aversion to advertisements in strategies of competing online services to mitigate the risk of leverage threat in platform markets. For instance, for a sufficiently small level of quality advantage, inferior quality firm can focus on reducing nuisance costs to counter leverage threat.

8.2 Policy Implications

The novel aspects of digital markets (e.g., free services with advertisements) are not well understood in terms of their welfare implications. Our model contributes to the debate on platform regulation by showing that a credible leverage of market power by a dominant firm, counter-intuitively, can be beneficial to consumers. This is especially likely in markets characterized by small to intermediate levels of differences in product qualities and large nuisance cost of advertisements. According to our model, this results from an increase in users' gross surplus and a decrease in total nuisance costs (because of fewer advertisements), thus increasing user welfare. However, for the same market, given the two-sided nature of services, a credible leverage of market power can have the opposite effect from a social point of view, decreasing advertisers' profit. Thus, a

[10] See, "Meta's Ray-Bans smart glasses sold more than 1 million units last year," January 2025, The Verge, available at https://www.theverge.com/meta/603674/meta-ray-ban-smart-glasses-sales.

regulation forcing dominant platforms to sell their products and services separately should be complemented with regulatory measures to reduce the disutility that users incur from viewing more advertisements to minimize consumer loss.

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Appendix

A Proofs of the Baseline Model

Proof of Lemma 1

First-Order Conditions

Under independent pricing, i.e., case NN, the profit of the firms are

$$\pi_G = p_{G1}N_{G1} + r_{G2}a_{G2} : \text{ Firm G's profit,} \tag{24}$$

$$\text{and } \pi_S = p_{S1}N_{S1} + r_{S2}a_{S2} : \text{ Firm S' profit.} \tag{25}$$

Since profit maximization for two components is independent of each other, we consider each component separately. First, consider hardware product. Since profit functions are continuously differentiable, any optimal pair of prices must satisfy the first-order necessary conditions of firms' optimization problem. Using Equations (24) and (25) and differentiating w.r.t. prices, they are

$$\frac{\partial \pi_G}{\partial p_{G1}} = N_{G1} + p_{G1}.\frac{\partial N_{G1}}{\partial p_{G1}} = 0, \text{ and } \frac{\partial \pi_S}{\partial p_{S1}} = N_{S1} + p_{S1}.\frac{\partial N_{S1}}{\partial p_{S1}} = 0. \tag{26}$$

Note that the first-order conditions defined by (26) must hold with equality since we allow for negative prices to be charged to the users. Solving them would give us equilibrium prices as defined in Lemma 1*(ii)*. They are

$$p^*_{G1} = \frac{3t + \Delta}{3}, \text{ and } p^*_{S1} = \frac{3t - \Delta}{3}. \tag{27}$$

Now, consider online service. Any optimal pair of advertising quantities must satisfy the first-order conditions of firms' optimization problem. Putting the value of $r_{i2} = (1 - a_{i2})N_{i2}, i = G, S$, (defined by Equation

(11)) in Equations (24) and (25), and differentiating w.r.t. advertising quantities, they are

$$\frac{\partial \pi_G}{\partial a_{G2}} = (1 - 2a_{G2})N_{G2} + (1 - a_{G2}).a_{G2}.\frac{\partial N_{G2}}{\partial a_{G2}} \leq 0, \text{ and} \tag{28}$$

$$\frac{\partial \pi_S}{\partial a_{S2}} = (1 - 2a_{S2})N_{S2} + (1 - a_{S2}).a_{S2}.\frac{\partial N_{S2}}{\partial a_{S2}} \leq 0. \tag{29}$$

First, we argue that the advertising levels in both firms are positive, which implies that the first-order conditions (28) and (29) bind. This follows since in any equilibrium with both firms having a positive market share, i.e., $0 < N_{G2}; N_{S2} < 1$, and zero advertising levels, i.e., $a^*_{G2} = a^*_{S2} = 0$, violate the first-order conditions (28) and (29). Thus, they bind, and at symmetric equilibrium, i.e., $a^*_{G2} = a^*_{S2} > 0$, gives us the advertising levels as defined in Lemma 1*(i)*. They are

$$a^*_{G2} = a^*_{S2} = \frac{1}{2} + \frac{t}{\delta} - \sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}}. \tag{30}$$

Second-Order Conditions

Next, we evaluate the Hessian matrix of each firm, denoted by $H_i, i = G, S$. We have

$$H_i = \begin{bmatrix} -\frac{1}{t} & 0 \\ 0 & -2N_{i2} - \frac{\delta}{t}.(1 - 2a_{i2}) \end{bmatrix}$$

From the preceding expression, we can see that the principal minor of order 1 is negative and algebraic calculations show that the principal minor of order 2 (i.e., determinant of H_i) is positive. Thus, principal minors alternate in sign and $H_i, i = G, S$, is negative semi-definite at the interior solution. This completes the proof. ■

Proof of Lemma 2

First-Order Conditions

As defined in the main text, under service integration regime, i.e., cases NI, IN, and II, $\tilde{y}$ denotes the average location of the indifferent user which is given by

$$\tilde{y} = \frac{x_1 + x_2}{2} = \frac{1}{2} + \frac{\Delta}{4t} + \frac{\tilde{p}_S - \tilde{p}_G}{4t} + \frac{\delta\tilde{a}_S - \delta\tilde{a}_G}{4t}. \tag{31}$$

Let $\tilde{F}(\cdot)$ and $\tilde{f}(\cdot)$ denote the distribution and probability density functions of the average location $\tilde{y}$. They are

$$\tilde{F}(y) = \begin{cases} 2y^2, & \text{if } 0 \leq y \leq \frac{1}{2}, \text{ and} \\ 1 - 2(1-y)^2, & \text{if } \frac{1}{2} < y \leq 1. \end{cases} \qquad \tilde{f}(y) = \begin{cases} 4y, & \text{if } 0 \leq y \leq \frac{1}{2}, \text{ and} \\ 4(1-y), & \text{if } \frac{1}{2} < y \leq 1. \end{cases} \tag{32}$$

The profit of the firms are

$$\widetilde{\pi}_G = \widetilde{p}_G\widetilde{N}_G + \widetilde{r}_G\widetilde{a}_G: \text{ Firm G's profit,} \tag{33}$$

$$\text{and } \widetilde{\pi}_S = \widetilde{p}_S\widetilde{N}_S + \widetilde{r}_S\widetilde{a}_S: \text{ Firm S' profit.} \tag{34}$$

Since profit functions are continuously differentiable, any optimal pair of prices and advertising quantities must satisfy the first-order conditions of firms' optimization problem. Putting the value of $\widetilde{r}_i = (1 - \widetilde{a}_i)\widetilde{N}_i, i = G, S$, (defined by Equation (18)) in Equations (33) and (34) and differentiating w.r.t. prices and advertising quantities, they are

$$\frac{\partial\widetilde{\pi}_G}{\partial\widetilde{p}_G} = \widetilde{N}_G + (\widetilde{p}_G + \widetilde{a}_G.(1 - \widetilde{a}_G))\frac{\partial\widetilde{N}_G}{\partial\widetilde{p}_G} = 0, \tag{35}$$

$$\frac{\partial\widetilde{\pi}_G}{\partial\widetilde{a}_G} = (1 - 2\widetilde{a}_G)\widetilde{N}_G + (\widetilde{p}_G + \widetilde{a}_G.(1 - \widetilde{a}_G))\frac{\partial\widetilde{N}_G}{\partial\widetilde{a}_G} \leq 0, \tag{36}$$

$$\frac{\partial\widetilde{\pi}_S}{\partial\widetilde{p}_S} = \widetilde{N}_S + (\widetilde{p}_S + \widetilde{a}_S.(1 - \widetilde{a}_S))\frac{\partial\widetilde{N}_S}{\partial\widetilde{p}_S} = 0, \text{ and} \tag{37}$$

$$\frac{\partial\widetilde{\pi}_S}{\partial\widetilde{a}_S} = (1 - 2\widetilde{a}_S)\widetilde{N}_S + (\widetilde{p}_S + \widetilde{a}_S.(1 - \widetilde{a}_S))\frac{\partial\widetilde{N}_S}{\partial\widetilde{a}_S} \leq 0. \tag{38}$$

First, consider the equilibrium advertising levels. If the advertising levels are positive, then first-order conditions (36) and (38) bind. Moreover, using first-order condition (36) together with first-order condition (35), and using first-order condition (38) together with first-order condition (37), gives

$$(1 - 2\widetilde{a}_i) = \delta, \quad i = G, S.$$

Solving the preceding equation gives us the equilibrium advertising level for firm $i, i = G, S$, as defined in Lemma 2*(i)*. Note that if $\delta \geq 1$, then $\widetilde{a}_G^* = \widetilde{a}_S^* = 0$.

Next, consider the equilibrium prices. Using first-order conditions (35) and (37), the equilibrium advertising levels, and the fact that $\widetilde{N}_G = \widetilde{F}(\widetilde{y})$, and $\partial\widetilde{N}_G/\partial\widetilde{p}_G$ = - $\widetilde{f}(\widetilde{y})/4t$, we obtain

$$\widetilde{p}_G^* = \frac{4t.\widetilde{F}(\widetilde{y})}{\widetilde{f}(\widetilde{y})} - \max\left\{\frac{(1 - \delta^2)}{4}, 0\right\}, \text{ and } \widetilde{p}_S^* = \frac{4t.(1 - \widetilde{F}(\widetilde{y}))}{\widetilde{f}(\widetilde{y})} - \max\left\{\frac{(1 - \delta^2)}{4}, 0\right\}. \tag{39}$$

From the preceding Equation (39), it can be seen that we need to solve for equilibrium value of $\widetilde{y}$ to obtain $\widetilde{p}_G^*$ and $\widetilde{p}_S^*$. Putting the value for $\widetilde{p}_S^* - \widetilde{p}_G^*$ and $\widetilde{a}_S^* - \widetilde{a}_G^*$ in Equation (31), we get

$$\widetilde{y} = \frac{1}{2} + \frac{\Delta}{4t} + \frac{1 - 2\widetilde{F}(\widetilde{y})}{\widetilde{f}(\widetilde{y})}. \tag{40}$$

In *Step i*, we establish the uniqueness and interiority of $\widetilde{y}^*$ and then, in *Step ii*, we derive the equilibrium $\widetilde{y}$.

Step i. There exists a unique and an interior $\widetilde{y}^*$, i.e., $1/2 < \widetilde{y}^* < 1$.

Consider the function $h(\widetilde{y})$ defined as

$$h(\widetilde{y}) = \frac{1}{2} + \frac{\Delta}{4t} + \frac{1 - 2\widetilde{F}(\widetilde{y})}{\widetilde{f}(\widetilde{y})} - \widetilde{y}. \tag{41}$$

From the preceding equation, and using Equation (32), we can see that $h(0) > 0$ and $h(1) \to -\infty$. Moreover,

$$\frac{dh(.)}{d\widetilde{y}} = \begin{cases} \frac{-1-8\widetilde{y}^2}{4\widetilde{y}^2} < 0 & \text{if } 0 \leq \widetilde{y} \leq 1/2, \text{ and} \\ \frac{-1-8(1-\widetilde{y})^2}{4(1-\widetilde{y})^2} < 0 & \text{if } 1/2 < \widetilde{y} \leq 1. \end{cases}$$

Therefore, using intermediate value theorem, there exists $\widetilde{y}^* \in (0, 1)$ such that $h(\widetilde{y}^*) = 0$, and (i) $h(\widetilde{y}) > 0$ for $0 \leq \widetilde{y} \leq \widetilde{y}^*$ and (ii) $h(\widetilde{y}) < 0$ for $\widetilde{y}^* < \widetilde{y} \leq 1$. Moreover, at $\widetilde{y} = 1/2$, we have $h(1/2) = \Delta/4t > 0$, implying that $\widetilde{y}^* \in (1/2, 1)$.

Step ii. Equilibrium $\widetilde{y}^*$.

Using $\widetilde{F}(\widetilde{y})$ and $\widetilde{f}(\widetilde{y})$ (defined by Equation (32)) in Equation (40), we get the value $\widetilde{y}^*$ by solving

$$\widetilde{y}^* = \frac{1}{2} + \frac{\Delta}{4t} + \frac{4(1 - \widetilde{y}^*)^2) - 1}{4(1 - \widetilde{y}^*)^2}.$$

This gives

$$\widetilde{y}^* = \frac{7t + \Delta/2 - \sqrt{9t^2 + \Delta^2/4 - t\Delta}}{8t}. \tag{42}$$

Now using the value of $\widetilde{y}^*$ in Equation (32), we get

$$\widetilde{f}(\widetilde{y}^*) = \frac{t - \Delta/2 + C}{2t}, \quad \text{and } \widetilde{F}(\widetilde{y}^*) = \frac{32t^2 - (t - \Delta/2 + C)^2}{32t^2}, \tag{43}$$

where $C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}$. Finally, using the preceding values in optimal price functions (defined by Equation (39)), we get the equilibrium prices as defined in Lemma 2*(ii)*. They are

$$\widetilde{p}_G^* = \frac{32t^2 - (t - \Delta/2 + C)^2}{4(t - \Delta/2 + C)} - \max\left\{\frac{(1 - \delta^2)}{4}, 0\right\}, \quad \text{and } \widetilde{p}_S^* = \frac{t - \Delta/2 + C}{4} - \max\left\{\frac{(1 - \delta^2)}{4}, 0\right\}.$$

Second-Order Conditions

Next, we evaluate the Hessian matrix of each firm, denoted by $H_i, i = G, S$. Using first-order conditions (defined by Equations (35)-(38)), and evaluating at optimal prices $\widetilde{p}_G^*$ and $\widetilde{p}_S^*$ and advertising levels $\widetilde{a}_G^* > 0$ and $\widetilde{a}_S^* > 0$ (defined in Lemma 2), the Hessian matrix is[11]

[11] For $\delta \geq 1$, we have $\widetilde{a}_i^* = 0$. Since the profit function is concave in $\widetilde{p}_i$, i.e., $\frac{\partial^2 \widetilde{\pi}_i}{\partial \widetilde{p}_i^2} = \frac{1}{4t}\left(\frac{\widetilde{N}_i(\partial \widetilde{f}(\cdot)/\partial \widetilde{y}) - 2\widetilde{f}(\cdot)^2}{\widetilde{f}(\cdot)}\right) < 0$, the second-order conditions hold for this case.

$$H_i = \begin{bmatrix} \frac{\partial^2 \widetilde{\pi}_i}{\partial \widetilde{p}_i^2} & \delta\left(\frac{\partial^2 \widetilde{\pi}_i}{\partial \widetilde{p}_i^2}\right) \\ \delta\left(\frac{\partial^2 \widetilde{\pi}_i}{\partial \widetilde{p}_i^2}\right) & \delta^2\left(\frac{\partial^2 \widetilde{\pi}_i}{\partial \widetilde{p}_i^2}\right) - 2\,\widetilde{N}_i \end{bmatrix},$$

where $\frac{\partial^2 \widetilde{\pi}_i}{\partial \widetilde{p}_i^2} = \frac{1}{4t}\left(\frac{\widetilde{N}_i(\partial \widetilde{f}(\cdot)/\partial \widetilde{y}) - 2\widetilde{f}(\cdot)^2}{\widetilde{f}(\cdot)}\right)$. Note that, using Equation (32), $\partial \widetilde{f}(\cdot)/\partial \widetilde{y} = -4$, for $\widetilde{y} \in (1/2, 1]$. This implies that $\frac{\partial^2 \widetilde{\pi}_i}{\partial \widetilde{p}_i^2} < 0$. Moreover, determinant of $H = \left(\frac{\partial^2 \widetilde{\pi}_i}{\partial \widetilde{p}_i^2}\right)\left[\delta^2\left(\frac{\partial^2 \widetilde{\pi}_i}{\partial \widetilde{p}_i^2}\right) - 2\widetilde{N}_i\right] - \left[\delta\left(\frac{\partial^2 \widetilde{\pi}_i}{\partial \widetilde{p}_i^2}\right)\right]^2 = -2\,\widetilde{N}_i\left(\frac{\partial^2 \widetilde{\pi}_i}{\partial \widetilde{p}_i^2}\right) > 0$. The principal minor of order 1 is negative, and the determinant of H_i is positive. Thus, principal minors alternate in sign, and $H_i, i = G, S$, is negative semi-definite at the interior solution. This completes the proof. ■

Proof of Proposition 1

First, consider firm G's optimal prices under the two regimes. Using Lemmas 1 and 2, the equilibrium prices under independent pricing and service integration are

$$p^*_{G1} = \frac{3t + \Delta}{3}, \quad \text{and} \quad \widetilde{p}^*_G = \frac{32t^2 - (t - \Delta/2 + C)^2}{4(t - \Delta/2 + C)} - \max\left\{\frac{(1-\delta^2)}{4}, 0\right\},$$

where $C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}$. Now, define $z(\Delta) = \widetilde{p}^*_G - p^*_{G1}$. In *Step i*, we show that $z(\Delta)$ is monotonically increasing in Δ, and in *Step ii*, we show that for $\delta \geq 1$, $z(\Delta) > 0$ for all $0 < \Delta < 3t$, and for $\delta < 1$, $z(\Delta)$ is negative for small values of Δ and positive otherwise.

Step i. $z(\Delta)$ is monotonically increasing in Δ.

Using values of $\widetilde{p}^*_G$, and p^*_{G1}, $z(\Delta)$ can be written as

$$z(\Delta) = \frac{32t^2 - (t - \Delta/2 + C)^2}{4(t - \Delta/2 + C)} - \max\left\{\frac{(1-\delta^2)}{4}, 0\right\} - \frac{(3t + \Delta)}{3}. \tag{44}$$

Taking derivative of $z(.)$ w.r.t. Δ gives

$$\frac{\partial z(.)}{\partial \Delta} = \left[\frac{8t^2}{(t - \Delta/2 + C)^2} + \frac{1}{4}\right]\left[\frac{1}{2} - \frac{(\Delta/2 - t)}{2\sqrt{9t^2 + \Delta^2/4 - t\Delta}}\right] - \frac{1}{3}.$$

Algebraic calculations show that the preceding expression is positive if $44t^2 - \Delta^2 + 4t\Delta + (t - \Delta/2 + C)^2 > 0$. Since $\Delta < 3t$ (Assumption 1), the preceding inequality always holds. Thus, $z(\Delta)$ is monotonically increasing in Δ.

Step ii. For $\delta \geq 1$, $z(\Delta) > 0$ for all $0 < \Delta < 3t$, and for $\delta < 1$, $z(\Delta)$ is negative for small values of Δ and positive otherwise.

Note that for $\delta \geq 1$, algebraic calculations show that $z(\Delta) > 0$, if $8t^2 > \left[t - \frac{\Delta}{2} + C\right]\left[\frac{5t}{4} + \frac{5\Delta}{24} + \frac{C}{4}\right]$, which is positive for all $\Delta > 0$, implying that $\widetilde{p}^*_G > p^*_{G1}$, for all $0 < \Delta < 3t$. Next, consider $\delta < 1$. As $\Delta \to 0$, we have $z(\Delta) \to -\frac{(1-\delta^2)}{4} < 0$. Whereas, when $\Delta \to 3t$, we have $z(\Delta) \to \frac{16t}{\sqrt{33}-1} - \frac{(\sqrt{33}-1)t}{8} - 2t - \frac{(1-\delta^2)}{4} \approx .778t - \frac{(1-\delta^2)}{4}$. It is positive if $t > \left(\frac{1-\delta^2}{3.112}\right) \approx 0.321(1-\delta^2)$. Given Assumption 2, the preceding inequality holds for all $\delta \geq 0$.

Hence, by intermediate value theorem, there exists a unique threshold $\Delta_{pG}(\delta) \in (0, 3t)$ such that $z(\Delta_{pG}(\delta)) = 0$. In other words, (i) for $0 < \Delta \leq \Delta_{pG}(\delta)$, we have $z(\Delta(\delta)) \leq 0$, implying that $\widetilde{p}_G^* \leq p_{G1}^*$, and (ii) for $\Delta_{pG}(\delta) < \Delta < 3t$, we have $z(\Delta) > 0$, implying that $\widetilde{p}_G^* > p_{G1}^*$.

Now, consider firm S' optimal prices under the two regimes. Using Lemmas 1 and 2, the equilibrium prices under independent pricing and service integration are

$$p_{S1}^* = \frac{3t - \Delta}{3}, \quad \text{and} \quad \widetilde{p}_S^* = \frac{t - \Delta/2 + C}{4} - \max\left\{\frac{(1-\delta^2)}{4}, 0\right\},$$

where $C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}$. Now, define $h(\Delta) = \widetilde{p}_S^* - p_{S1}^*$. In *Step i*, we show that $h(\Delta)$ is monotonically increasing in Δ. In *Step ii*, we show that for $\delta \geq 1$, $h(\Delta) > 0$ for all $0 < \Delta < 3t$, and for $\delta < 1$, $h(\Delta)$ is negative for small values of Δ and positive otherwise.

Step i. $h(\Delta)$ is monotonically increasing in Δ.

Define $h(\Delta) = \widetilde{p}_S^* - p_{S1}^*$. Using values of $\widetilde{p}_S^*$, and p_{S1}^*, it can be written as

$$h(\Delta) = \frac{t - \Delta/2 + C}{4} - \max\left\{\frac{(1-\delta^2)}{4}, 0\right\} - \frac{(3t - \Delta)}{3}. \tag{45}$$

Taking derivative of $h(.)$ w.r.t. Δ gives

$$\frac{\partial h(.)}{\partial \Delta} = -\frac{1}{4}\left[\frac{1}{2} - \frac{(\Delta/2 - t)}{2\sqrt{9t^2 + \Delta^2/4 - t\Delta}}\right] + \frac{1}{3}.$$

Algebraic calculations show that the preceding expression is positive if $216t^2 + 4\Delta^2 - 16t\Delta > 0$. Since $\Delta < 3t$ (Assumption 1), this always holds and thus, $h(\Delta)$ is monotonically increasing in Δ.

Step ii. For $\delta \geq 1$, $h(\Delta) > 0$, for all $0 < \Delta < 3t$, and for $\delta < 1$, $h(\Delta)$ is negative for small values of Δ and positive otherwise.

To begin, consider $\delta \geq 1$. Algebraic calculations show that $h(\Delta) > 0$, if $t - \frac{\Delta}{9} > 0$ which holds given Assumption 1, implying that $\widetilde{p}_S^* > p_{S1}^*$ for all $0 < \Delta < 3t$. Next consider $\delta < 1$. As $\Delta \to 0$, we have $h(\Delta) \to -(1-\delta^2)/4 < 0$. Whereas, as $\Delta \to 3t$, $h(\Delta) \to (\sqrt{33}-1)t/8 - (1-\delta^2)/4$. Given Assumption 2, it is positive.

Hence, by intermediate value theorem, there exists a unique threshold $\Delta_{pS}(\delta) \in (0, 3t)$, such that $h(\Delta_{pS}(\delta)) = 0$. In other words, (i) for $0 < \Delta \leq \Delta_{pS}(\delta)$, we have $h(\Delta) \leq 0$, implying that $\widetilde{p}_S^* \leq p_{S1}^*$, and (ii) for $\Delta_{pS}(\delta) < \Delta < 3t$, we have $h(\Delta) > 0$, implying that $\widetilde{p}_S^* > p_{S1}^*$.

Next, we show that $\Delta_{pG}(\delta) < \Delta_{pS}(\delta)$. We know that $\widetilde{p}_S^*(\Delta_{pS}(\delta)) = p_{S1}^*(\Delta_{pS}(\delta))$. Now, evaluating $z(\Delta) = \widetilde{p}_G^* - p_{G1}^*$ (defined by Equation (44)) at $\Delta_{pS}(\delta)$ and using the equality $\widetilde{p}_S^*(\Delta_{pS}(\delta)) = p_{S1}^*(\Delta_{pS}(\delta))$, we have

$$z(\Delta_{pS}) = \frac{8t^2}{(t - \Delta_{pS}/2 + C)} - \frac{(t - \Delta_{pS}/2 + C)}{2} - \frac{2\Delta_{pS}}{3}.$$

Note that we have suppressed the argument δ and written $z(\Delta_{pS}(\delta))$ as $z(\Delta_{pS})$ for simplicity. Using the value of $C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}$, algebraic calculations show that the preceding expression is positive if

$$\left[18t^2 + \frac{\Delta_{pS}^2}{2} + 2t\Delta_{pS}\right]^2 > \left[9t^2 + \frac{\Delta_{pS}^2}{4} - t\Delta_{pS}\right][6t + \Delta_{pS}],$$

which always holds. Thus, using the fact that $z(\Delta)$ is monotonically increasing in Δ and $z(\Delta_{pG}(\delta)) = 0$, it must be that $\Delta_{pG}(\delta) < \Delta_{pS}(\delta)$ for all $\delta \geq 0$.

Next, we compare the advertising levels under the two regimes. Let us define $a_{G2}^* = a_{S2}^* = a^*$, and $\widetilde{a}_G^* = \widetilde{a}_S^* = \widetilde{a}^*$. Using Lemmas 1 and 2, they are

$$a^* = \frac{1}{2} + \frac{t}{\delta} - \sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}}, \quad \text{and} \quad \widetilde{a}^* = \max\left\{\frac{(1-\delta^2)}{4}, 0\right\}.$$

From the preceding equation it can be seen that if $\delta \geq 1$, then $a^* > \widetilde{a}^*$ always. If $\delta < 1$, then a^* is greater than $\widetilde{a}^*$ depending on t and δ. We can find a threshold $t_a(\delta)$ where

$$t_a(\delta) = \frac{\delta(3 - 2\delta^2 - \delta^4)}{8(1 + \delta^2)},$$

such that (i) for $t \leq t_a(\delta)$, $a^* \leq \widetilde{a}^*$ and (ii) for $t > t_a(\delta)$, $a^* > \widetilde{a}^*$. Note that $\max_{\delta \in [0,1)} t_a(\delta) < \frac{1}{2}$. Thus, given Assumption 2, we have $a^* > \widetilde{a}^*$. This completes the proof. ■

Proof of Proposition 2

First, we compare firm G's profit under independent pricing and service integration. Using Lemmas 1 and 2, the equilibrium profits under the two regimes are

$$\pi_G^* = \frac{(3t + \Delta)^2}{18t} + \frac{t}{\delta}\left[\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}} - \frac{t}{\delta}\right], \quad \text{and} \quad \widetilde{\pi}_G^* = \frac{[32t^2 - (t - \Delta/2 + C)^2]^2}{128t^2(t - \Delta/2 + C)},$$

where $C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}$. Now, define $g(\Delta) = \widetilde{\pi}_G^* - \pi_G^*$. In *Step i*, we show that $g(\Delta)$ is monotonically increasing in Δ. In *Step ii*, we show that $g(\Delta)$ is negative for small values of Δ and positive otherwise.

Step i. $g(\Delta)$ is monotonically increasing in Δ.

It can be written as

$$g(\Delta) = \frac{8t^2}{(t-\Delta/2+C)} + \frac{(t-\Delta/2+C)^3}{128t^2} - \frac{(t-\Delta/2+C)}{2} - \frac{(3t+\Delta)^2}{18t} - \frac{t}{\delta}\left[\sqrt{\frac{1}{4}+\frac{t^2}{\delta^2}} - \frac{t}{\delta}\right]. \quad (46)$$

Taking the derivative of the preceding expression w.r.t. Δ gives

$$\frac{\partial g(\Delta)}{\partial \Delta} = \frac{8t^2}{2C(t-\Delta/2+C)} - \frac{3(t-\Delta/2+C)^3}{256t^2C} + \frac{(t-\Delta/2+C)}{4C} - \frac{(3t+\Delta)}{9t}. \quad (47)$$

Algebraic calculations show that the preceding expression is positive if

$$9t[32t^2-(t-\Delta/2+C)^2][32t^2+3(t-\Delta/2+C)^2]-(3t+\Delta)(256Ct^2)(t-\Delta/2+C)>0$$

Given $\Delta < 3t$ (Assumption 1) and Assumption 2, the preceding inequality always holds. Thus, $g(\Delta)$ is monotonically increasing in Δ.[12]

Step ii. $g(\Delta)$ is negative for small values of Δ and positive otherwise.

For $\Delta \to 0$, $g(\Delta) = -\frac{t}{\delta}\left[\sqrt{\frac{1}{4}+\frac{t^2}{\delta^2}} - \frac{t}{\delta}\right]$. Whereas, for $\Delta \to 3t$, we have $g(\Delta) \approx 0.2904t - \frac{t}{\delta}\left[\sqrt{\frac{1}{4}+\frac{t^2}{\delta^2}} - \frac{t}{\delta}\right]$. It is positive if $t > 0.430 - 0.1452\delta^2$. Since $\delta \in [0,1)$, and given Assumption 2, this always holds. Hence, by intermediate value theorem, there exists a unique $\Delta_G(\delta) \in (0, 3t)$ such that $g(\Delta_G(\delta)) = 0$. In other words, (i) for $0 < \Delta \le \Delta_G(\delta)$, we have $g(\Delta) \le 0$, implying that $\tilde{\pi}^*_G \le \pi^*_G$ and (ii) for $\Delta_G(\delta) < \Delta < 3t$, we have $g(\Delta) > 0$, implying that $\tilde{\pi}^*_G > \pi^*_G$. Moreover, since π^*_G is strictly decreasing in δ, the derivative $\Delta'_G(\delta) < 0$.

Next, consider firm S' profit. Using Lemmas 1 and 2, the equilibrium profits under the two regimes are

$$\pi^*_S = \frac{(3t-\Delta)^2}{18t} + \frac{t}{\delta}\left[\sqrt{\frac{1}{4}+\frac{t^2}{\delta^2}} - \frac{t}{\delta}\right], \text{ and } \tilde{\pi}^*_S = \frac{(t-\Delta/2+C)^3}{128t^2},$$

where $C = \sqrt{9t^2+\Delta^2/4-t\Delta}$. In *Step i*, we show that $s(\Delta)$ is monotonically increasing for small Δ and decreasing otherwise, and in *Step ii*, $s(\Delta)$ is negative for small values of Δ and positive otherwise.

Step i. $s(\Delta)$ is strictly concave in Δ with monotonically increasing for small Δ and decreasing otherwise.

It can be written as

$$s(\Delta) = \frac{(t-\Delta/2+C)^3}{128t^2} - \frac{(3t-\Delta)^2}{18t} - \frac{t}{\delta}\left[\sqrt{\frac{1}{4}+\frac{t^2}{\delta^2}} - \frac{t}{\delta}\right]. \quad (48)$$

Taking the derivative of the preceding expression w.r.t. Δ gives

$$\frac{\partial s(\Delta)}{\partial \Delta} = -\frac{3(t-\Delta/2+C)^3}{256Ct^2} + \frac{(3t-\Delta)}{9t}. \quad (49)$$

For $\Delta \to 0$, we have $\partial s(\Delta)/\partial\Delta = 1/12$. Whereas, for $\Delta \to 3t$, we have $\partial s(\Delta)/\partial\Delta = -\frac{0.156t}{C}$. Moreover,

$$\frac{\partial^2 s(\Delta)}{\partial \Delta^2} = 3\left[\frac{(t-\Delta/2+C)^3}{512C^2t^2}\right]\left[\frac{(t-\Delta/2-2C)}{C}\right] - \frac{1}{9t} < 0,$$

because $2C > t - \frac{\Delta}{2}$. This implies that $s(\Delta)$ is strictly concave in Δ and $\partial s(\Delta)/\partial\Delta$ is strictly positive for small Δ and strictly negative otherwise.

[12] Moreover, we also conduct numerical simulations with $t \ge 1.2$, and find that the preceding expression is positive for all $\Delta > 0$.

Step ii. $s(\Delta)$ is negative for small Δ and positive otherwise.

We show this step in two parts.

Part a. As $\Delta \to 3t$, $s(\Delta) > 0$.

For $\Delta \to 3t$, we have $s(\Delta) = .104t - \frac{t}{\delta}\left[\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}} - \frac{t}{\delta}\right]$. Given Assumption 2, algebraic calculations show that the preceding expression is positive for all $\delta \geq 0$.

Part b. There exists a unique $\Delta_S(\delta) \in (0, 3t)$ such that $s(\Delta) \leq 0$ for $0 < \Delta \leq \Delta_S(\delta)$ and is positive otherwise.

Suppose not, i.e., there exists $\Delta_S(\delta), \Delta_{S'}(\delta) \in (0, 3t)$ such that $s(\Delta_S(\delta)) = s(\Delta_{S'}(\delta)) = 0$. Also, w.l.o.g., assume that $\Delta_S(\delta) < \Delta_{S'}(\delta)$. Then, using the fact that $s(\Delta) < 0$ as $\Delta \to 0$ and $s(\Delta)$ is strictly concave in Δ (*Step i*) we can argue that

(bi) for $0 < \Delta \leq \Delta_S(\delta)$, we have $\tilde{\pi}_S^*(\Delta) \leq \pi_S^*(\Delta)$,

(bii) for $\Delta_S(\delta) < \Delta \leq \Delta_{S'}(\delta)$, we have $\tilde{\pi}_S^*(\Delta) > \pi_S^*(\Delta)$, and

(biii) for $\Delta_{S'}(\delta) < \Delta < 3t$, we have $\tilde{\pi}_S^*(\Delta) < \pi_S^*(\Delta)$.

This contradicts our claim from part (a) that $\tilde{\pi}_S^*(\Delta) > \pi_S^*(\Delta)$ as $\Delta \to 3t$. Thus, there exists a unique $\Delta_S(\delta) \in (0, 3t)$ such that $\tilde{\pi}_S^*(\Delta_S(\delta)) = \pi_S^*(\Delta_S(\delta))$. In other words, (a) for $0 < \Delta \leq \Delta_S(\delta)$, we have $s(\Delta) \leq 0$ implying that $\tilde{\pi}_S^*(\Delta) \leq \pi_S^*(\Delta)$, and (b) for $\Delta_S(\delta) < \Delta < 3t$, we have $s(\Delta) > 0$ implying that $\tilde{\pi}_S^*(\Delta) > \pi_S^*(\Delta)$. Moreover, since π_S^* is strictly decreasing in δ, the derivative $\Delta_S'(\delta) < 0$.

Next, we argue that $\Delta_G(\delta) < \Delta_S(\delta)$. We know that $\tilde{\pi}_S^*(\Delta_S(\delta)) = \pi_S^*(\Delta_S(\delta))$. Now, evaluating $g(\Delta) = \tilde{\pi}_G^* - \pi_G^*$ defined by Equation (46) at $\Delta_S(\delta)$ and using the equality $\tilde{\pi}_S^*(\Delta_S(\delta)) = \pi_S^*(\Delta_S(\delta))$, we have

$$g(\Delta_S) = \frac{8t^2}{(t - \Delta_S/2 + C)} - \frac{(t - \Delta_S/2 + C)}{2} - \frac{2\Delta_S}{3}.$$

Note that we have suppressed the argument δ and written $g(\Delta_S(\delta))$ as $g(\Delta_S)$ for simplicity. Using the value of $C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}$, algebraic calculations show that the preceding expression is positive if

$$\left[18t^2 + \frac{\Delta_S^2}{2} + 2t\Delta_S\right]^2 > \left[9t^2 + \frac{\Delta_S^2}{4} - t\Delta_S\right][6t + \Delta_S],$$

which always holds. Thus, using the fact that $g(\Delta)$ is monotonically increasing in Δ and $g(\Delta_G(\delta)) = 0$, it must be that $\Delta_G(\delta) < \Delta_S(\delta)$ for all $\delta \geq 0$. This completes the proof. ■

Proof of Proposition 4

First, we examine the change in user welfare. Consider the independent pricing regime, i.e., case NN. When $0 < N_{Gi}^*, N_{Si}^* < 1, i = 1, 2$, then user welfare is

$$UW^* = \int_0^{x_1^*} [V_{G1} - p_{G1}^* - tx_1]dx_1 + \int_{x_1^*}^1 [V_{S1} - p_{S1}^* - t(1 - x_1)]dx_1 + \int_0^{x_2^*} [W - \delta a_{G2}^* - tx_2]\, dx_2 + \int_{x_2^*}^1 [W - \delta a_{S2}^* - t(1 - x_2)]\, dx_2.$$

Using the *Stage 2* equilibrium values defined in Lemma 1, the preceding equation equals

$$UW^* = \frac{V_{G1} + V_{S1}}{2} + \frac{5\Delta^2}{36t} - \frac{t}{2} + W - \frac{(3t+\Delta)^2}{18t} - \frac{(3t-\Delta)^2}{18t} - \delta a^*, \tag{50}$$

where a^* is as defined in Lemma 1*(i)*.

Next, consider service integration regime, i.e., cases NI, IN, and II. User welfare is

$$\widetilde{UW}^* = \int_0^{1/2} [V_{G1} + W - \tilde{p}^*_G - \delta\tilde{a}^*_{G2} - ty]\, 4y\; dy + \int_{1/2}^{\tilde{y}^*} [V_{G1} + W - \tilde{p}^*_G - \delta\tilde{a}^*_{G2} - ty]\, 4(1-y)\; dy$$
$$+ \int_{\tilde{y}^*}^{1} [V_{S1} + W - \tilde{p}^*_S - \delta\tilde{a}^*_{S2} - t(1-y)]\, 4(1-y)\; dy.$$

Using the *Stage 2* equilibrium values defined in Lemma 2, the preceding equation equals

$$\widetilde{UW}^* = V_{G1} + W - 2(1-\tilde{y}^*)^2\Delta - \frac{7t}{6} + \frac{8t(\tilde{y}^*)^3}{3} - 6t(\tilde{y}^*)^2 + 4t\tilde{y}^* + 2(1-\tilde{y}^*)^2(\tilde{p}^*_G - \tilde{p}^*_S) - \tilde{p}^*_G - \delta\tilde{a}^*, \tag{51}$$

where $\tilde{y}^*$ is as defined by Equation (42), and $\tilde{p}^*_G$, $\tilde{p}^*_S$, and $\tilde{a}^*$ are as defined in Lemma 2.

Using Equations (50) and (51), the change in user welfare, i.e., $\Delta UW = \widetilde{UW}^* - UW^*$ is

$$\Delta UW = \Delta\left[\frac{1}{2} - 2(1-\tilde{y}^*)^2\right] - \frac{5\Delta^2}{36t} - \frac{2t}{3} + \frac{8t(\tilde{y}^*)^3}{3} - 6t(\tilde{y}^*)^2 + 4t\tilde{y}^*$$
$$+ 2(1-\tilde{y}^*)^2(\tilde{p}^*_G - \tilde{p}^*_S) - \tilde{p}^*_G + \frac{(3t+\Delta)^2}{18t} + \frac{(3t-\Delta)^2}{18t} - \delta(\tilde{a}^* - a^*). \tag{52}$$

First, consider the case when Δ is very small, i.e., Δ is close to 0. Then, using Equation (42), we can see that $\tilde{y}^* \to 1/2$. Also, using Lemma 2*(ii)*, $\tilde{p}^*_G = \tilde{p}^*_S$. Using this value in Equation (52) gives $\Delta UW \to \frac{7t}{6} - \tilde{p}^*_G - \delta(\tilde{a}^* - a^*)$. We can consider two sub-cases: case (a): $\delta \geq 1$ and case (b): $\delta < 1$. Under case (a), $\tilde{a}^* = 0$ and $\tilde{p}^*_G = t$, implying that $\Delta UW \to \frac{t}{6} + \delta a^* > 0$. Under case (b), $\tilde{a}^* = \frac{1-\delta}{2}$ and $\tilde{p}^*_G = t - \frac{(1-\delta^2)}{4}$, implying that $\Delta UW \to \frac{t}{6} + \delta a^* + \frac{(1-\delta)^2}{4} > 0$.

Next, consider the case when Δ is very large, i.e., Δ is close to $3t$. Under independent pricing regime, $x_1^* \to 1$, implying user welfare will be $UW^* = V_{S1} + \Delta + W - \frac{5t}{4} - \delta a^*$. Under service integration regime, using Equation (42), we can see that $\tilde{y}^* \to 1$. This gives $\widetilde{UW}^* = V_{S1} + \Delta + W - \frac{t}{2} - \delta\tilde{a}^* - \tilde{p}^*_G$. Using these values, $\Delta UW \to \frac{3t}{4} - \tilde{p}^*_G - \delta(\tilde{a}^* - a^*)$. We can consider two sub-cases: case (a): $\delta \geq 1$, and case (b): $\delta < 1$. Under case (a), $\tilde{a}^* = 0$ and using Lemma 2*(ii)*, $\tilde{p}^*_G = 2.78t$, implying $\Delta UW \to \delta a^* - 2.03t$. Using the value of a^* (as defined in Lemma 1*(i)*), implies that $\delta a^* - 2.03t < 0$ if $\frac{\delta}{2} + t - \delta\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}} < 2.03t$. This requires $\left[\frac{\delta}{2} - 1.03t\right]^2 < \frac{\delta^2}{4} + t^2$, which always holds. Hence, $\Delta UW < 0$. Under case (b), $\tilde{a}^* = \frac{1-\delta}{2}$ and $\tilde{p}^*_G = 2.78t - \frac{1-\delta^2}{4}$, implying that $\Delta UW \to \delta a^* - 2.03t + \frac{(1-\delta)^2}{4}$. Using value of a^* (as defined in Lemma 1*(i)*), $\delta a^* - 2.03t + \frac{(1-\delta)^2}{4} < 0$, if $\frac{1+\delta^2}{4} < 1.03t + \delta\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}}$. Given Assumption 2, $1.03t + \delta\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}} > \frac{1+\delta^2}{4}$, and thus $\Delta UW < 0$.

Now, using $\tilde{p}^*_G = \frac{4t\tilde{f}(\tilde{y}^*)}{\tilde{f}(\tilde{y}^*)} = t\left[\frac{1}{1-\tilde{y}^*} - 2(1-\tilde{y}^*)\right]$, and $\tilde{p}^*_G - \tilde{p}^*_S = \frac{t[1-4(1-\tilde{y}^*)^2]}{1-\tilde{y}^*}$ in Equation (52), and taking derivative with respect to Δ, gives

$$\frac{\partial(\Delta UW)}{\partial\Delta} = \frac{1}{2} - 2(1-\widetilde{y}^*)^2 - \frac{\Delta}{18t} + \left[16t(1-\widetilde{y}^*)^2 + 4\Delta(1-\widetilde{y}^*) + 8t(\widetilde{y}^*)^2 - 12t\widetilde{y}^* - \frac{t}{(1-\widetilde{y}^*)^2}\right]\frac{\partial\widetilde{y}^*}{\partial\Delta}. \tag{53}$$

Note that $\frac{\partial\widetilde{y}^*}{\partial\Delta} > 0$. Moreover, algebraic calculations show that as $\Delta \to 0$, $\frac{\partial(\Delta UW)}{\partial\Delta} \to -4t\frac{\partial\widetilde{y}^*}{\partial\Delta}$, and as $\Delta \to 3t$, $\frac{\partial(\Delta UW)}{\partial\Delta} \to -\infty$. For the case with $0 < \Delta < 3t$, we conduct a numerical analysis with $t = 1.2$, and find that $\frac{\partial(\Delta UW)}{\partial\Delta} < 0$. Therefore, $\frac{\partial(\Delta UW)}{\partial\Delta} < 0$ for all $0 < \Delta < 3t$. Hence, by intermediate value theorem, there exists a threshold $\Delta_{uw}(\delta)$ with $\Delta'_{uw}(\delta) > 0$ such that for (i) for $0 < \Delta \le \Delta_{uw}(\delta)$, $\Delta UW \ge 0$, and (ii) for $\Delta_{uw}(\delta) < \Delta < 3t$, $\Delta UW < 0$.

As an illustration, Table 1 below highlights the change in user welfare for $t = 1.202$; three values of Δ, i.e., $\Delta = 0.2$, $\Delta = 1.5$ and $\Delta = 2.9$; and three values of δ, i.e., $\delta = 0.2$, $\delta = 0.9$ and $\delta = 3.5$. As Table 1 shows, for small to intermediate Δ, nuisance costs decrease sufficiently and, jointly with the rise in users' gross surplus, can increase user welfare. Whereas, for large Δ, increase in prices and total transportation costs dominate the gain in users' gross surplus from consuming the system G1G2 and fall in total nuisance costs, decreasing user welfare.

Values	$\Delta = 0.2$	$\Delta = 1.5$	$\Delta = 2.9$
$\delta = 0.2$	0.3	- 0.15	- 0.7
$\delta = 0.9$	0.4	- 0.1	- 0.5
$\delta = 3.5$	0.8	0.1	- 0.4

Table 1: Change in user welfare

Next, consider advertisers' profit (AP). Under the independent pricing regime, it is

$$AP^* = \int_{\alpha^*}^{1} [\alpha N^*_{G2} - r^*_{G2}]d\alpha + \int_{\alpha^*}^{1} [\alpha N^*_{S2} - -r^*_{S2}]d\alpha.$$

Using the *Stage 2* equilibrium values for advertising price, i.e., $r^*_{i2} = (1-a^*)$, where $i = G, S$, and a^* is as defined in Lemma 1*(i)*, the preceding equation equals

$$AP^* = \frac{(a^*)^2}{2}. \tag{54}$$

Next, consider the service integration regime. Similarly, we can define advertisers' profit as

$$\widetilde{AP}^* = \frac{(\widetilde{a}^*)^2}{2}, \tag{55}$$

where $\widetilde{a}^*$ is as defined in Lemma 2*(i)*. Since $\widetilde{a}^* < a^*$, we obtain $\widetilde{AP}^* < AP^*$. This completes the proof. ■

B Extensions and Robustness Checks

B.1 Free Online Services with No Advertisements

Proof of Proposition 3

In this extension, we consider the scenario when there are no advertisements in the services. First, consider independent pricing, i.e. case NN. The profit of the firms are

$$\pi_G = p_{G1}N_{G1}: \text{ Firm G's profit, and}$$

$$\pi_S = p_{S1}N_{S1}: \text{ Firm S' profit,}$$

where $N_{i1}, i = G, S$, is as defined by Equation (9) in the baseline model. Following an approach similar to that in the proof of Lemma 1, we can derive equilibrium prices. They are

$$p^*_{G1} = \frac{3t+\Delta}{3}, \text{ and } p^*_{S1} = \frac{3t-\Delta}{3}.$$

This gives profit of the firms as

$$\pi^*_G = \frac{(3t+\Delta)^2}{18t}, \text{ and } \pi^*_S = \frac{(3t-\Delta)^2}{18t}. \tag{56}$$

Next, consider the service integration regime, i.e. cases IN, NI, and II. The profit of the firms are

$$\widetilde{\pi}_G = \widetilde{p}_G\widetilde{N}_G: \text{ Firm G's profit, and} \tag{57}$$

$$\widetilde{\pi}_S = \widetilde{p}_S\widetilde{N}_S: \text{ Firm S' profit,} \tag{58}$$

where $\widetilde{N}_G = \widetilde{F}(\widetilde{y})$ and $\widetilde{N}_S = 1 - \widetilde{F}(\widetilde{y})$. Setting $\delta = 0$ in Equation (31), and using $\widetilde{F}(\widetilde{y})$ (defined by Equation (32)), we can obtain the value of $\widetilde{y}$ as defined by Equation (40). Now, following the approach used in the proof of Lemma 2, we can derive equilibrium prices. They are

$$\widetilde{p}^*_G = \frac{32t^2 - (t-\Delta/2+C)^2}{4(t-\Delta/2+C)}, \text{ and } \widetilde{p}^*_S = \frac{t-\Delta/2+C}{4},$$

where $C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}$. Using these prices in Equations (32) and (40), we can derive equilibrium demands. This gives equilibrium profits as

$$\widetilde{\pi}^*_G = \frac{[32t^2 - (t-\Delta/2+C)^2]^2}{128t^2(t-\Delta/2+C)}, \text{ and } \widetilde{\pi}^*_S = \frac{(t-\Delta/2+C)^3}{128t^2},$$

where $C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}$. Next, we compare the equilibrium profits under the two regimes. First, we compare firm G's profit. Let $g(\Delta) = \widetilde{\pi}^*_G - \pi^*_G$. After some algebra, it can be defined as

$$g(\Delta) = \frac{8t^2}{(t-\Delta/2+C)} + \frac{(t-\Delta/2+C)^3}{128t^2} - \frac{(t-\Delta/2+C)}{2} - \frac{(3t+\Delta)^2}{18t}.$$

Taking the derivative of the preceding expression w.r.t. Δ gives

$$\frac{\partial g(\Delta)}{\partial \Delta} = \frac{8t^2}{2C(t-\Delta/2+C)} - \frac{3(t-\Delta/2+C)^3}{256t^2C} + \frac{(t-\Delta/2+C)}{4C} - \frac{(3t+\Delta)}{9t}.$$

As argued in the proof of Proposition 2, the preceding expression is positive for all $0 < \Delta < 3t$. At $\Delta \to 0$, $g(\Delta) = 0$, whereas for $\Delta \to 3t$, we have $g(\Delta) = 0.3t > 0$. Therefore, $g(\Delta) > 0$ for all $0 < \Delta < 3t$. Next, consider firm S' profit. Let $s(\Delta) = \widetilde{\pi}^*_S - \pi^*_S$. After some algebra, it can be defined as

$$s(\Delta) = \frac{(t-\Delta/2+C)^3}{128t^2} - \frac{(3t-\Delta)^2}{18t}.$$

Taking the derivative of the preceding expression w.r.t. Δ gives

$$\frac{\partial s(\Delta)}{\partial \Delta} = -\frac{3(t-\Delta/2+C)^3}{256Ct^2} + \frac{(3t-\Delta)}{9t}.$$

As argued in the proof of Proposition 2, $s(\Delta)$ is strictly concave in Δ, and $\partial s(\Delta)/\partial\Delta$ is strictly positive for small Δ and strictly negative otherwise. At $\Delta \to 0$, $s(\Delta) = 0$, whereas for $\Delta \to 3t$, we have $s(\Delta) = 0.104t > 0$. Therefore, we can argue that $s(\Delta) > 0$ for all $0 < \Delta < 3t$. Thus, we have $\widetilde{\pi}_i^* > \pi_i^*, i = G, S$, for all $0 < \Delta < 3t$. This completes the proof. ■

B.2 Asymmetric Intensity of Competition

First, we consider independent pricing regime, i.e. case NN. At *Stage 4*, users make participation decisions. For the product, the user indifferent between consuming product G1 and S1 is defined by the location $\hat{x}_1 \in (0,1)$, such that $V_{G1} - p_{G1} - t_1\hat{x}_1 = V_{S1} - p_{S1} - t_1(1-\hat{x}_1)$, yielding $\hat{x}_1 = \frac{1}{2} + \frac{\Delta}{2t_1} + \frac{p_{S1}-p_{G1}}{2t_1}$, where $\Delta = V_{G1} - V_{S1} > 0$, by assumption. This gives demand for product G1 as $N_{G1} = \hat{x}_1 = \frac{1}{2} + \frac{\Delta}{2t_1} + \frac{p_{S1}-p_{G1}}{2t_1}$, and for product S1 as $N_{S1} = 1 - \hat{x}_1 = \frac{1}{2} - \frac{\Delta}{2t_1} + \frac{p_{G1}-p_{S1}}{2t_1}$. For the online service, the user indifferent between consuming service G2 and S2 is defined by the location $\hat{x}_2 \in (0,1)$, such that $W - \delta a_{G2} - t_2\hat{x}_2 = W - \delta a_{S2} - t_2(1-\hat{x}_2)$, yielding $\hat{x}_2 = \frac{1}{2} + \frac{\delta a_{S2}-\delta a_{G2}}{2t_2}$. This gives the demand for service G2 as $N_{G2} = \hat{x}_2 = \frac{1}{2} + \frac{\delta a_{S2}-\delta a_{G2}}{2t_2}$, and demand for service S2 as $N_{S2} = 1 - \hat{x}_2 = \frac{1}{2} + \frac{\delta a_{G2}-\delta a_{S2}}{2t_2}$. At *Stage 3*, the inverse advertising demand function of firm $i, i = G, S$, is $r_{i2} = (1 - a_{i2})N_{i2}, i = G, S$. Using the inverse advertising demand function and the user demand functions in the firms' profit functions defined by Equations (1) and (2), firm $i, i = G, S$, chooses the user price p_{i1} and advertising quantity a_{i2} to maximize its profits. The *Stage 2* equilibrium satisfies the following:

(i) The equilibrium advertising quantities are $a_{G2}^* = a_{S2}^* = \frac{1}{2} + \frac{t_2}{\delta} - \sqrt{\frac{1}{4} + \left(\frac{t_2}{\delta}\right)^2}$, and equilibrium product prices are $p_{G1}^* = \frac{3t_1+\Delta}{3}$ and $p_{S1}^* = \frac{3t_1-\Delta}{3}$,

(ii) The equilibrium market shares are $N_{G1}^* = \frac{3t_1+\Delta}{6t_1}$, $N_{S1}^* = \frac{3t_1-\Delta}{6t_1}$, and $N_{S2}^* = N_{G2}^* = \frac{1}{2}$.

(iii) The equilibrium profit of the firms are

$$\pi_G^* = \frac{(3t_1+\Delta)^2}{18t_1} + \frac{t_2}{\delta}\left[\sqrt{\frac{1}{4} + \left(\frac{t_2}{\delta}\right)^2} - \frac{t_2}{\delta}\right], \text{ and } \pi_S^* = \frac{(3t_1-\Delta)^2}{18t_1} + \frac{t_2}{\delta}\left[\sqrt{\frac{1}{4} + \left(\frac{t_2}{\delta}\right)^2} - \frac{t_2}{\delta}\right]. \tag{59}$$

Under service integration, i.e. cases NI, IN, and II, the indifferent user with location (x_1, x_2) is given by $V_{G1} + W - \delta\tilde{a}_G - \tilde{p}_G - (t_1x_1 + t_2x_2) = V_{S1} + W - \delta\tilde{a}_S - \tilde{p}_S - (t_1(1-x_1) + t_2(1-x_2))$. Let $\tilde{y}$ denote the weighted average location of the indifferent user which is given by

$$\tilde{y} = \frac{t_1x_1 + t_2x_2}{2(t_1+t_2)} = \frac{1}{2} + \frac{\Delta}{2(t_1+t_2)} + \frac{\tilde{p}_S - \tilde{p}_G}{2(t_1+t_2)} + \frac{\delta\tilde{a}_S - \delta\tilde{a}_G}{2(t_1+t_2)}. \tag{60}$$

Let $\widetilde{F}(.)$ and $\widetilde{f}(.)$ denote the distribution and probability density functions of the average location $\tilde{y}$. They are

$$\widetilde{F}(y)=\begin{cases}\frac{(t_1+t_2)^2}{2t_1t_2}y^2, & \text{if } 0\le y\le \frac{t_2}{t_1+t_2},\\ \frac{(t_1+t_2)y}{t_1}-\frac{(t_1+t_2)^2}{2t_1t_2}\left(\frac{t_2}{t_1+t_2}\right)^2, & \text{if } \frac{t_2}{t_1+t_2}<y\le\frac{t_1}{t_1+t_2}, \text{ and}\\ 1-\frac{(t_1+t_2)^2}{2t_1t_2}(1-y)^2, & \text{if } \frac{t_1}{t_1+t_2}<y\le 1.\end{cases} \tag{61}$$

and

$$\widetilde{f}(y)=\begin{cases}\frac{(t_1+t_2)^2}{t_1t_2}y, & \text{if } 0\le y\le \frac{t_2}{t_1+t_2},\\ \frac{(t_1+t_2)}{t_1}, & \text{if } \frac{t_2}{t_1+t_2}<y\le\frac{t_1}{t_1+t_2}, \text{ and}\\ \frac{(t_1+t_2)^2}{t_1t_2}(1-y), & \text{if } \frac{t_1}{t_1+t_2}<y\le 1.\end{cases} \tag{62}$$

The profit of the firms are

$$\widetilde{\pi}_G=\widetilde{p}_G\widetilde{N}_G+\widetilde{r}_G\widetilde{a}_G: \text{ Firm G's profit,} \tag{63}$$

$$\text{and } \widetilde{\pi}_S=\widetilde{p}_S\widetilde{N}_S+\widetilde{r}_S\widetilde{a}_S: \text{ Firm S' profit.} \tag{64}$$

Using $\widetilde{N}_G=\widetilde{F}(\widetilde{y})$, and $\widetilde{N}_S=1-\widetilde{F}(\widetilde{y})$, and $\widetilde{r}_i=(1-\widetilde{a}_i)\widetilde{N}_i, i=G,S$, in Equations (63) and (64) and differentiating w.r.t. prices and advertising quantities, they are

$$\frac{\partial\widetilde{\pi}_G}{\partial\widetilde{p}_G}=\widetilde{N}_G+(\widetilde{p}_G+\widetilde{a}_G.(1-\widetilde{a}_G))\frac{\partial\widetilde{N}_G}{\partial\widetilde{p}_G}=0, \tag{65}$$

$$\frac{\partial\widetilde{\pi}_G}{\partial\widetilde{a}_G}=(1-2\widetilde{a}_G)\widetilde{N}_G+(\widetilde{p}_G+\widetilde{a}_G.(1-\widetilde{a}_G))\frac{\partial\widetilde{N}_G}{\partial\widetilde{a}_G}\le 0, \tag{66}$$

$$\frac{\partial\widetilde{\pi}_S}{\partial\widetilde{p}_S}=\widetilde{N}_S+(\widetilde{p}_S+\widetilde{a}_S.(1-\widetilde{a}_S))\frac{\partial\widetilde{N}_S}{\partial\widetilde{p}_S}=0, \text{ and} \tag{67}$$

$$\frac{\partial\widetilde{\pi}_S}{\partial\widetilde{a}_S}=(1-2\widetilde{a}_S)\widetilde{N}_S+(\widetilde{p}_S+\widetilde{a}_S.(1-\widetilde{a}_S))\frac{\partial\widetilde{N}_S}{\partial\widetilde{a}_S}\le 0. \tag{68}$$

First, consider the equilibrium advertising levels. Using first-order condition (66) together with first-order condition (65), and first-order condition (68) together with first-order condition (67), yields $(1-2\widetilde{a}_i)=\delta, \quad i=G,S.$. This gives $\widetilde{a}_i^*=\frac{1-\delta}{2}, i=G,S$. Note that if $\delta\ge 1$, then $\widetilde{a}_G^*=\widetilde{a}_S^*=0$. Next, consider the equilibrium prices. Using first-order conditions (65) and (67), the equilibrium advertising levels, and the fact that $\widetilde{N}_G=\widetilde{F}(\widetilde{y})$, and $\frac{\partial\widetilde{N}_G}{\partial\widetilde{p}_G}=-\frac{\widetilde{f}(\widetilde{y})}{2(t_1+t_2)}$, we obtain

$$\widetilde{p}_G^*=\frac{2(t_1+t_2)\widetilde{F}(\widetilde{y})}{\widetilde{f}(\widetilde{y})}-\max\left\{\frac{(1-\delta^2)}{4},0\right\}, \text{ and } \widetilde{p}_S^*=\frac{2(t_1+t_2)(1-\widetilde{F}(\widetilde{y}))}{\widetilde{f}(\widetilde{y})}-\max\left\{\frac{(1-\delta^2)}{4},0\right\}. \tag{69}$$

Putting the value for $\widetilde{p}_S^*-\widetilde{p}_G^*$ and $\widetilde{a}_S^*-\widetilde{a}_G^*$ in Equation (60), we get

$$\widetilde{y}=\frac{1}{2}+\frac{\Delta}{2(t_1+t_2)}+\frac{1-2\widetilde{F}(\widetilde{y})}{\widetilde{f}(\widetilde{y})}.$$

If $\frac{t_2}{t_1+t_2} \leq \widetilde{y}^* < \frac{t_1}{t_1+t_2}$,[13] then

$$\widetilde{y}^* = \frac{6t_2 + \Delta}{12t_2}. \tag{70}$$

Numerical analysis with values $V = 1.5, W = 1.5,\ t_1 = 1.2, t_2 = 0.9,\ \delta \in [0, 5]$ and $\Delta \in [0, 3.6]$, shows that for $\Delta \in [0, 0.77)$, $\frac{t_2}{t_1+t_2} < \frac{6t_2+\Delta}{12t_2} < \frac{t_1}{t_1+t_2}$, and $\frac{t_1}{t_1+t_2} \leq \frac{6t_2+\Delta}{12t_2} < 1$, otherwise.

If $\widetilde{y}^* > \frac{t_1}{t_1+t_2}$, then

$$\widetilde{y}^* = \frac{7t_2 + \Delta/2 - \sqrt{9(t_2)^2 + \Delta^2/4 - t_2\Delta}}{8t_2}. \tag{71}$$

Numerical analysis with values $V = 1.5, W = 1.5,\ t_1 = 1.2, t_2 = 0.9,\ \delta \in [0, 5]$ and $\Delta \in [0, 3.6]$, shows that for $\Delta \in [0, 0.77)$, $\frac{t_2}{t_1+t_2} < \frac{7t_2+\Delta/2-\sqrt{9(t_2)^2+\Delta^2/4-t_2\Delta}}{8t_2} < \frac{t_1}{t_1+t_2}$, and $\frac{t_1}{t_1+t_2} \leq \frac{7t_2+\Delta/2-\sqrt{9(t_2)^2+\Delta^2/4-t_2\Delta}}{8t_2} < 1$, otherwise.

Now using the value of $\widetilde{y}^*$, $\widetilde{f}(\widetilde{y}^*)$, and $\widetilde{F}(\widetilde{y}^*)$ in Equation (69), we get the equilibrium prices. The equilibrium profits are

$$\widetilde{\pi}_G^* = \frac{2(t_1 + t_2)\widetilde{F}(\widetilde{y}^*)}{\widetilde{f}(\widetilde{y}^*)}, \text{ and } \widetilde{\pi}_S^* = \frac{2(t_1 + t_2)(1 - \widetilde{F}(\widetilde{y}^*))}{\widetilde{f}(\widetilde{y}^*)}. \tag{72}$$

Next, we rely on numerical analysis to examine the impact of service integration on firms' profit. We conduct numerical analysis with values $V = 1.5, W = 1.5,\ t_1 = 1.2, t_2 = 0.9,\ \delta \in [0, 5]$ and $\Delta \in [0, 3.6]$, with $\widetilde{y}^* = \frac{6t_2+\Delta}{12t_2}$ for $\Delta \in [0, 0.77]$, and $\widetilde{y}^* = \frac{7t_2+\Delta/2-\sqrt{9(t_2)^2+\Delta^2/4-t_2\Delta}}{8t_2}$ for $\Delta \in [0.77, 3.6]$, and the corresponding expressions of $\widetilde{F}(\widetilde{y}^*)$, and $\widetilde{f}(\widetilde{y}^*)$ defined by Equations (61) and (62), respectively. The numerical results are illustrated in Figure 2. The numerical analysis shows that our main results remain unchanged for sufficiently large values of δ.[14] ■

B.3 Multi-Homing Users

First, consider independent pricing regime, i.e. case NN. For the hardware product, a user's net utility is as defined by Equation (5). For the online service, a user's net utility from single-homing and consuming either service G2 or S2 is defined by Equation (6). Whereas, if a user multi-homes and consumes both services, then her utility is

$$2W - \delta a_{S2} - \delta a_{G2} - t. \tag{73}$$

At *Stage 4*, users make participation decisions. For the product, user demand functions are defined by Equation (9). For the online service, using Equations (6) and (73), the user indifferent between consuming only G2 (single-homing) and and multi-homing is located at x_1, such that (a) $U(x_1, G2) = U(x_1, G2S2)$, (b) $U(x, G2) \geq U(x, G2S2)$, for all $x \leq x_1$, and (c) $U(x, G2) \leq U(x, G2S2)$, for all $x \geq x_1$, where

[13] To bring out main insights clearly, we consider values of t_1 and t_2 such that $t_1 > t_2$.

[14] Please note that we have done multiple robustness checks by varying the set of parameter values. In all numerical experiments, we find that our insights are the same.

$x_1 = 1 - \frac{[W - \delta a_{S2}]}{t}$. Similarly, using Equations (6) and (73), the user indifferent between consuming only S2 (single-homing) and multi-homing is located at x_2, such that (a) $U(x_2, S2) = U(x_2, G2S2)$, (b) $U(x, S2) \geq U(x, G2S2)$, for all $x \geq x_2$, and (c) $U(x, S2) \leq U(x, G2S2)$, for all $x \leq x_2$, where $x_2 = \frac{W - \delta a_{G2}}{t}$. A necessary condition for multi-homing in market B is $N_{G2} + N_{S2} \geq 1$, implying that $x_1 \leq x_2$. Using the values of x_1 and x_2, this requires $a_{G2} + a_{S2} \leq \frac{2W-1}{\delta}$. It then follows that firms' demands for services are $N_{G2} = \frac{W - \delta a_{G2}}{t}$, and $N_{S2} = \frac{W - \delta a_{S2}}{t}$. At *Stage 3*, the inverse advertising demand function of firm $i, i = G, S$, is given by $r_{i2} = (1 - a_{i2})N_{i2}, i = G, S$. Using the inverse advertising demand function, the user demand functions for products and services in the firms' profit functions defined by Equations (1) and (2), firm $i, i = G, S$, chooses the user price p_{i1} and advertising quantity a_{i2} to maximize its profits. This yields the following:

(i) The equilibrium prices are $p^*_{G1} = \frac{3t+\Delta}{3}$, and $p^*_{S1} = \frac{3t-\Delta}{3}$.

(ii) The equilibrium advertising levels are $a^*_{G2} = a^*_{S2} = \frac{\delta + W - \sqrt{\delta^2 - \delta W + W^2}}{3\delta}$.

(iii) The equilibrium profit of the firms are

$$\pi^*_G = \frac{(3t+\Delta)^2}{18t} + \frac{1}{2}(1 - a^*_{G2})a^*_{G2}, \text{ and } \pi^*_S = \frac{(3t-\Delta)^2}{18t} + \frac{1}{2}(1 - a^*_{S2})a^*_{S2}.$$

Next, we consider the service integration regime, i.e. cases IN, NI, and II. Since users are restricted to choose between two systems G1G2 and S1S2, the *Stage 2* equilibrium remains the same as defined in Lemma 2.

Note that since equilibrium prices remain unchanged, the price comparison remains the same as defined in Proposition 1. Next, we compare advertising levels under the two regimes. For $\delta \geq 1$, we have $\tilde{a}^* = 0$, thus $a^* > \tilde{a}^*$. For $\delta < 1$, $a^* > \tilde{a}^*$ requires $\frac{\delta + W - \sqrt{\delta^2 - \delta W + W^2}}{3\delta} > \frac{(1-\delta)}{2}$. Define $f(W, \delta) = \frac{\delta + W - \sqrt{\delta^2 - \delta W + W^2}}{3\delta} - \frac{(1-\delta)}{2}$. First, note that at $W = \frac{1}{2}$, algebraic calculations show that $f(W, \delta) > 0$, requires $\frac{9\delta^4}{4} + 1.5\delta(1-\delta) > 0$. which always hold for $\delta < 1$. Also, it can be shown that $f(W, \delta)$ is increasing in W. Since multi-homing requires $W > \frac{1}{2}$, we have $f(W, \delta) > 0$, thus $a^* > \tilde{a}^*$. Therefore, advertising levels decrease with service integration. Next, we rely on numerical analysis to examine the impact of service integration on firms' profit. We conduct numerical analysis with values $W = 1.5$, $t = 1.2$, $\delta \in (0, 5]$ and $\Delta \in [0, 3.6]$, and compare firms' profit under two market regimes as shown in Figure 3. The numerical analysis shows that our main results remain unchanged for sufficiently large values of δ. ■

B.4 Competition in Advertising Prices

In the baseline model, at *Stage 2*, we considered advertising competition between firms by treating advertising quantities per user as the strategic variable and derived firms' advertising revenue/profit under different market regimes. Equivalently, we can treat "*price per ad per viewer*" as the strategic variable. Suppose an advertiser pays per viewer price r_{i2} (respectively, $\tilde{r}_i$) under independent pricing (respectively, service integration). Then an advertiser α will place an advertisement in firm i's service under independent pricing (respectively, service integration) if $\alpha \geq r_{i2}$ (respectively, $\alpha \geq \widetilde{r}_i$), and the number of advertisements placed will be $1 - r_{i2}$ (respectively, $1 - \widetilde{r}_i$) under independent pricing (respectively, service integration). A firm's advertising profit can be reinterpreted as per viewer revenue times the number of viewers. Thus, firm i's total profit will be

$$\pi_i = p_{i1}N_{i1} + r_{i2}(1 - r_{i2})N_{i2} : \text{ Independent Pricing,} \tag{74}$$

$$\text{and } \widetilde{\pi}_i = \widetilde{p}_i\widetilde{N}_i + \widetilde{r}_i(1 - \widetilde{r}_i)\widetilde{N}_i : \text{ Service Integration.} \tag{75}$$

The second component of firm i's profit function means that under independent pricing (respectively, service integration) $1 - r_{i2}$ (respectively, $1 - \widetilde{r}_i$) advertisers pay a per viewer price r_{i2} (respectively, $\widetilde{r}_i$) to firm i whenever N_{i2} (respectively, $\widetilde{N}_i$) users click and view an advertisement. Now, at *Stage 2*, firms compete in user prices and advertising prices per viewer to maximize its profits under the different market regimes. At *Stage 1*, firms simultaneously decide whether to adopt independent pricing or service integration. The following proposition summarizes the main result.

Proposition 6. *When either firm G or firm S or both can adopt service integration, then the following holds:*

(i) For a sufficiently small level of firm G's quality advantage, both firms adopt independent pricing, i.e., case NN is an equilibrium.

(ii) For an intermediate level of firm G's quality advantage, there is service integration because firm G adopts it, whereas firm S adopts independent pricing, i.e., case IN is an equilibrium.

(iii) For a sufficiently large level of firm G's quality advantage, there is service integration because both firms adopt it, i.e., case II is an equilibrium.

Proof of Proposition 6: With per viewer advertising price, an advertiser α payoff function is

$$\pi_\alpha = \begin{cases} (\alpha - r_{i2})N_{i2} : & \text{Independent pricing, and} \\ (\alpha - \widetilde{r}_i)\widetilde{N}_i : & \text{Service Integration.} \end{cases} \tag{76}$$

Now, using the preceding expression, the number of advertisements in service $i2, i = G, S$, under independent pricing (case NN) is $a_{i2} = 1 - r_{i2}$, whereas under service integration (cases NI, IN, and II), it is $\widetilde{a}_i = 1 - \widetilde{r}_i$.

First, consider independent pricing, i.e. case NN. The profit of the firms are

$$\pi_G = p_{G1}N_{G1} + r_{i2}(1 - r_{G2})N_{G2} : \text{ Firm G's profit,} \tag{77}$$

$$\text{and } \pi_S = p_{S1}N_{S1} + r_{S2}(1 - r_{S2})N_{S2} : \text{ Firm S' profit,} \tag{78}$$

where $N_{i1}, i = G, S$, and $N_{i2}, i = G, S$, are defined by Equations (9) and (10) in the baseline model. Since profit maximization for the hardware product and online service is independent of each other, we consider each component separately.

First, consider the hardware product. Since profit functions are continuously differentiable, any optimal pair of prices must satisfy the first-order necessary conditions of firms' optimization problem. Using Equations (77) and (78) and differentiating w.r.t. prices, they are

$$\frac{\partial \pi_G}{\partial p_{G1}} = N_{G1} + p_{G1}.\frac{\partial N_{G1}}{\partial p_{G1}} = 0, \text{ and } \frac{\partial \pi_S}{\partial p_{S1}} = N_{S1} + p_{S1}.\frac{\partial N_{S1}}{\partial p_{S1}} = 0.$$

Solving the preceding first-order conditions, gives equilibrium prices as defined in Lemma 1*(ii)*. They are $p^*_{G1} = \frac{3t+\Delta}{3}$, and $p^*_{S1} = \frac{3t-\Delta}{3}$. Now, consider online service. Any optimal pair of per-viewer advertising prices must satisfy the first-order necessary conditions of the firms' optimization problem. They are

$$\frac{\partial \pi_G}{\partial r_{G2}} = (1-2r_{G2})N_{G2}+(1-r_{G2}).r_{G2}.\frac{\partial N_{G2}}{\partial a_{G2}}\frac{\partial a_{G2}}{\partial r_{G2}} \le 0, \text{ and } \frac{\partial \pi_S}{\partial r_{S2}} = (1-2r_{S2})N_{S2}+(1-r_{S2}).r_{S2}.\frac{\partial N_{S2}}{\partial a_{S2}}\frac{\partial a_{S2}}{\partial r_{S2}} \le 0.$$

At symmetric equilibrium, i.e. $r^*_{G2} = r^*_{S2} > 0$ would give us the advertising prices

$$r^*_{G2} = r^*_{S2} = \frac{1}{2} - \frac{t}{\delta} + \sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}}.$$

This gives advertising quantities as

$$a^*_{G2} = a^*_{S2} = \frac{1}{2} + \frac{t}{\delta} - \sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}},$$

which is the same as defined in Lemma 1*(i)* in the baseline model. Thus, equilibrium profits also remain unchanged (as defined in Lemma 1*(i)*).

Next, consider service integration, i.e. cases IN, NI, and II. The profit of the firms are

$$\widetilde{\pi}_G = \widetilde{p}_G \widetilde{N}_G + \widetilde{r}_G(1-\widetilde{r}_G)\widetilde{N}_G : \text{ Firm G's profit,}$$

$$\text{and } \widetilde{\pi}_S = \widetilde{p}_S \widetilde{N}_S + \widetilde{r}_S(1-\widetilde{r}_S)\widetilde{N}_S : \text{ Firm S' profit,}$$

where $\widetilde{N}_G = \widetilde{F}(\widetilde{y})$ and $\widetilde{N}_S = 1 - \widetilde{F}(\widetilde{y})$. The value of $\widetilde{y}$ is as defined by Equation (40) and $\widetilde{F}(\widetilde{y})$ is as defined by Equation (32). Since profit functions are continuously differentiable, any optimal pair of user prices and advertising prices must satisfy the first-order necessary conditions of firms' optimization problem. They are

$$\frac{\partial \widetilde{\pi}_G}{\partial \widetilde{p}_G} = \widetilde{N}_G + (\widetilde{p}_G + \widetilde{r}_G.(1-\widetilde{r}_G))\frac{\partial \widetilde{N}_G}{\partial \widetilde{p}_G} = 0, \tag{79}$$

$$\frac{\partial \widetilde{\pi}_G}{\partial \widetilde{r}_G} = (1-2\widetilde{r}_G)\widetilde{N}_G + (\widetilde{p}_G + \widetilde{r}_G.(1-\widetilde{r}_G))\frac{\partial \widetilde{N}_G}{\partial \widetilde{a}_G}\frac{\partial \widetilde{a}_G}{\partial \widetilde{r}_G} \le 0, \tag{80}$$

$$\frac{\partial \widetilde{\pi}_S}{\partial \widetilde{p}_S} = \widetilde{N}_S + (\widetilde{p}_S + \widetilde{r}_S.(1-\widetilde{r}_S))\frac{\partial \widetilde{N}_S}{\partial \widetilde{p}_S} = 0, \text{ and} \tag{81}$$

$$\frac{\partial \widetilde{\pi}_S}{\partial \widetilde{r}_S} = (1-2\widetilde{r}_S)\widetilde{N}_S + (\widetilde{p}_S + \widetilde{r}_S.(1-\widetilde{r}_S))\frac{\partial \widetilde{N}_S}{\partial \widetilde{a}_S}\frac{\partial \widetilde{a}_S}{\partial \widetilde{r}_S} \le 0. \tag{82}$$

Equation (80) together with Equation (79) and Equation (82) together with Equation (81) gives $(2\widetilde{r}_i - 1) = \delta, \ \ i = G, S$. Solving the preceding equation gives us

$$\widetilde{r}^*_i = \frac{\delta+1}{2}.$$

This gives equilibrium advertising as

$$\widetilde{a}_i^* = \max\left\{\frac{1-\delta}{2}, 0\right\},$$

which is the same as defined in Lemma 2*(i)* in the baseline model. Therefore, we get the same equilibrium prices as defined in Lemma 2*(ii)*. They are

$$\widetilde{p}_G^* = \frac{32t^2-(t-\Delta/2+C)^2}{4(t-\Delta/2+C)} - \max\left\{\frac{1-\delta^2}{4}, 0\right\}, \text{ and } \widetilde{p}_S^* = \frac{t-\Delta/2+C}{4} - \max\left\{\frac{1-\delta^2}{4}, 0\right\},$$

where $C = \sqrt{9t^2+\Delta^2/4-t\Delta}$. Thus, equilibrium profits remain unchanged (as defined in Lemma 2*(ii)*). Since equilibrium profits under both independent pricing and service integration remain unchanged, our main result on profit comparison will also be the same, as stated in Proposition 2. This completes the proof.

■

B.5 Pay-Per Click Advertising Model

In the baseline model, we assumed that an advertiser pays an access fee for advertising in firm i's service, $i = G, S$, irrespective of whether or not a user clicks and purchases the product. This might be at deviation from the pricing scheme in certain industries. For instance, "per click pricing" under which search engines collect fees from advertisers every time a consumer clicks on their link. However, this fact can be easily introduced by considering a slight variant of our model. Suppose an advertiser α pays the price r_{i2} (respectively, $\widetilde{r}_i$) under independent pricing (respectively, service integration) for advertising in service $i2, i = G, S$, only if a user clicks on its advertisement. Then, under this interpretation, an advertiser α expected profit is

$$\pi_\alpha = \begin{cases} (\alpha - r_{i2})N_{i2} : & \text{Independent Pricing, and} \\ (\alpha - \widetilde{r}_i)\widetilde{N}_i : & \text{Service Integration.} \end{cases} \tag{83}$$

At *Stage 3*, advertisers make the participation decision. An advertiser would advertise in service $i2, i = G, S$, as long as the marginal benefit of an advertisement is greater than its marginal cost. Using Equation (83), the inverse advertising demand function of firm $i, i = G, S$, under independent pricing (case NN) is $r_{i2} = 1 - a_{i2}$, whereas under service integration (cases NI, IN, and II) it is $\widetilde{r}_i = 1 - \widetilde{a}_i$. Firm i's profit is

$$\pi_i = p_{i1}N_{i1} + a_{G2}N_{i2}r_{i2} : \text{ Independent Pricing,} \tag{84}$$

$$\text{and } \widetilde{\pi}_i = \widetilde{p}_i\widetilde{N}_i + \widetilde{a}_G\widetilde{N}_i\widetilde{r}_i : \text{ Service Integration.} \tag{85}$$

The second component of firm i's profit function means that a_{i2} (respectively, $\widetilde{a}_i$) advertisers at firm i pay r_{i2} (respectively, $\widetilde{r}_i$) whenever N_{i2} (respectively, $\widetilde{N}_i$) users click on an advertisement. Next, using the inverse advertising demand functions, the user demand functions defined by Equations (9) and (10) in the baseline model, and by Equation (32) for service integration and putting the values for them in the profit functions defined by Equations (84) and (85), firm $i, i = G, S$, chooses the user price and advertising quantity to maximize its profits under the different market regimes at *Stage 2*. At *Stage 1*, firms simultaneously decide whether to adopt independent pricing or service integration. The following proposition summarizes the main findings.

Proposition 7. *When either firm G or firm S or both can adopt service integration, then the following holds:*

(i) For a sufficiently small level of firm G's quality advantage, both firms adopt independent pricing, i.e., case NN is an equilibrium,

(ii) For an intermediate level of firm G's quality advantage, there is service integration because firm G adopts it, whereas firm S adopts independent pricing, i.e., case IN is an equilibrium, and

(iii) For a sufficiently large level of firm G's quality advantage, there is service integration because both firms adopt it, i.e., case II is an equilibrium.

The preceding proposition shows that the main result on market equilibrium remains unchanged.

Proof of Proposition 7

With pay-per-click pricing, the advertiser α payoff function is

$$\pi_\alpha = \begin{cases} (\alpha - r_{i2})N_{i2} : & \text{Independent pricing, and} \\ (\alpha - \widetilde{r}_i)\widetilde{N}_i : & \text{Service Integration.} \end{cases} \tag{86}$$

Now, using the preceding expression, the inverse advertising demand function of firm $i, i = G, S$, under independent pricing (case NN) is $r_{i2} = 1 - a_{i2}$, whereas under service integration (cases NI, IN, and II) it is $\widetilde{r}_i = 1 - \widetilde{a}_i$. Now, using these demand functions in firm i's profit function defined by Equations (84), and (85), we can obtain firm i's profit function as

$$\pi_i = p_{i1}N_{i1} + a_{i2}N_{i2}r_{i2} : \text{ Independent Pricing,} \tag{87}$$

$$\text{and } \widetilde{\pi}_i = \widetilde{p}_i\widetilde{N}_i + \widetilde{a}_i\widetilde{N}_i\widetilde{r}_i : \text{ Service Integration,} \tag{88}$$

where $N_{i1}, i = G, S$, and $N_{i2}, i = G, S$, are as defined by Equations (9) and (10) in the baseline model, and $\widetilde{N}_G = \widetilde{F}(\widetilde{y})$ and $\widetilde{N}_S = 1 - \widetilde{F}(\widetilde{y})$. The value of $\widetilde{y}$ is as defined by Equation (40) and $\widetilde{F}(\widetilde{y})$ is as defined by Equation (32). Maximizing the preceding profit functions w.r.t. prices and advertising quantities would lead to the equilibrium prices, advertising levels, and profits as defined in Lemmas 1 and 2 in the main text. Since equilibrium prices, advertising levels, and profits remain unchanged, our key results remain unchanged. This completes the proof. ■

B.6 Auctions for Advertising Slots

In the baseline model, we did not consider the auction for advertising slots. However, we can set up a slight variant of our model to accommodate platforms selling impressions via second-price auctions.

Suppose there is a unit mass of homogeneous advertisers trying to reach users through placing ads in online services. The firms set up an auction to sell ad impressions on each of their consumers. Let the expected benefit of an advertiser α from purchasing m_{i2} (respectively, $\widetilde{m}_i$) advertising slots per user under independent pricing (respectively, service integration) in service $i2, i = G, S$, be

$$\pi_\alpha = \begin{cases} \alpha(1 - m_i)m_i N_{i2} : & \text{Independent pricing, and} \\ \alpha(1 - \widetilde{m}_i)\widetilde{m}_i \widetilde{N}_i : & \text{Service Integration,} \end{cases} \tag{89}$$

where α is uniform across advertisers, and for simplicity set equal to 1. Given m_{i2} (respectively, $\widetilde{m}_i$) under independent pricing (respectively, service integration), all advertisers have identical expected benefits. This implies that all advertisers will submit a symmetric bid. In equilibrium, the bid submitted by an advertiser equals its expected benefit from purchasing m_i advertising slots in service i. It equals $(1-m_i)m_i N_{i2}$, under independent pricing regime, i.e, case NN, and $(1 - \widetilde{m}_i)\widetilde{m}_i\widetilde{N}_i$, under service integration regime, i.e., cases IN, NI, and II. Each firm specifies a total quantity a_i (respectively, $\widetilde{a}_i$), i.e. the total number of advertising slots in its service. Since advertisers are homogeneous, the revenue functions are concave (defined by Equation (89)) and users are single-homing, each firm sets $m_{i2} = a_{i2}$ (respectively, $\widetilde{m}_i = \widetilde{a}_i$) to maximize its profits, where a_{i2} (respectively, $\widetilde{a}_i$) is the number of advertising slots per user sold to a single advertiser under independent pricing (respectively, service integration). This implies that each advertiser will purchase $m_{i2} = a_{i2}$ (respectively, $\widetilde{m}_i = \widetilde{a}_i$) slots under independent pricing (respectively, service integration). As a result, the firms' profit functions under different market regimes will remain unchanged. Under independent pricing, profit functions will be the same as defined by Equations (24) and (25). Similarly, under service integration, profit functions will be the same as defined by Equations (33) and (34). Therefore, firm i's profit maximization at *Stage 2* will remain unchanged, and thus, we will obtain equilibrium prices, advertising levels, and profits as defined in Lemmas 1 and 2. As a result, at *Stage 1*, the profit comparison will be the same as in Proposition 2. This completes the proof. ■

B.7 Application Developers

In the baseline model, we did not explicitly model for the application developers' side. In one of the main motivating examples, services are application stores. They earn revenue from search advertising and the sale of applications to the users. We now extend the model to include this additional source of revenue. Suppose there is a homogeneous mass one of application developers available on each service, i.e. all application developers multi-home. A user's utility when consuming service $i2, i = G, S$, is

$$\begin{cases} W + (b - s)D_{G2} - \delta a_{G2} - tx_2, & \text{if she consumes service G2, and} \\ W + (b - s)D_{S2} - \delta a_{S2} - t(1 - x_2), & \text{if she consumes service S2,} \end{cases} \tag{90}$$

where D_{i2} is the number of applications available on service $i2$, b is the benefit from consuming each application, and s is the price paid for each application. Under our assumption that $b \geq s$, $D_{i2} = 1, i = G, S$. Let $W' = W + b - s$. Each application developer pays a constant commission rate γ to a firm for each sale transacted through their service platforms. The rest of the model remains the same as described in the baseline model. The following proposition states our main result on the profit comparison.

Proposition 8. *When either firm G or firm S or both can adopt service integration, then the following holds:*

(i) Both firms adopt independent pricing for a small level of quality advantage.

(ii) Firm G adopts service integration, whereas firm S adopts independent pricing for an intermediate level of quality advantage.

(iii) Both firms adopt service integration for a sufficiently large level of quality advantage.

(iv) Profit difference between service integration and independent pricing regimes is (a) increasing in δ, and (b) decreasing in γ and s.

Proof of Proposition 8

Consider independent pricing, i.e. case NN. The profit of the firms are

$$\pi_G = p_{G1}N_{G1} + r_{G2}a_{G2} + \gamma s N_{G2} : \text{ Firm G's profit,}$$

$$\text{and } \pi_S = p_{S1}N_{S1} + r_{S2}a_{S2} + \gamma s N_{S2} : \text{ Firm S' profit,}$$

where $r_{i2} = (1 - a_{i2})N_{i2}$ is as defined by Equation (11) in the baseline model, $N_{i1}, i = G, S$, and $N_{i2}, i = G, S$, are as defined by Equations (9) and (10) in the baseline model. Following a similar approach as in the proof of Lemma 1, we can obtain the equilibrium prices, advertising quantities, market shares and profits. They are:

i. The equilibrium advertising levels are characterized by

$$a^*_{G2} = a^*_{S2} = \frac{1}{2} + \frac{t}{\delta} - \sqrt{\frac{1}{4} + \frac{t^2}{\delta^2} + \gamma s}.$$

ii. The equilibrium prices, and market shares are

$$p^*_{G1} = \frac{3t + \Delta}{3}, \; p^*_{S1} = \frac{3t - \Delta}{3}, \text{ and}$$

$$N^*_{G1} = \frac{3t + \Delta}{6t}, \; N^*_{S1} = \frac{3t - \Delta}{6t}, \; N^*_{S2} = N^*_{G2} = \frac{1}{2}.$$

The equilibrium profit of the firms are

$$\pi^*_G = \frac{(3t + \Delta)^2}{18t} + \frac{t}{\delta}\left[\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2} + \gamma s} - \frac{t}{\delta}\right], \text{ and } \pi^*_S = \frac{(3t - \Delta)^2}{18t} + \frac{t}{\delta}\left[\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2} + \gamma s} - \frac{t}{\delta}\right].$$

Next, consider the service integration regime, i.e. cases IN, NI, and II. The profit of the firms are

$$\widetilde{\pi}_G = \widetilde{p}_G \widetilde{N}_G + \widetilde{r}_G \widetilde{a}_G + \gamma s \widetilde{N}_G : \text{ Firm G's profit,}$$

$$\text{and } \widetilde{\pi}_S = \widetilde{p}_S \widetilde{N}_S + \widetilde{r}_S \widetilde{a}_S + \gamma s \widetilde{N}_S : \text{ Firm S' profit,}$$

where $\widetilde{r}_i$ is as defined by Equation (18) in the baseline model, $\widetilde{N}_i, i = G, S$, is obtained using Equations (40) and (32). Following a similar approach as in the proof of Lemma 2, we can obtain the equilibrium prices, advertising quantities, market shares and profits. They are:

i. The equilibrium advertising levels are

$$\widetilde{a}_G^* = \widetilde{a}_S^* = \max\left\{\frac{1-\delta}{2}, 0\right\}.$$

ii. The equilibrium prices and market shares are

$$\widetilde{p}_G^* = \frac{32t^2 - (t - \Delta/2 + C)^2}{4(t - \Delta/2 + C)} - \max\left\{\frac{1-\delta^2}{4}, 0\right\} - \gamma s,$$

$$\widetilde{p}_S^* = \frac{t - \Delta/2 + C}{4} - \max\left\{\frac{1-\delta^2}{4}, 0\right\} - \gamma s,$$

$$\widetilde{N}_G^* = \frac{32t^2 - (t - \Delta/2 + C)^2}{32t^2}, \text{ and } \widetilde{N}_S^* = \frac{(t - \Delta/2 + C)^2}{32t^2}, \text{ with } C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}.$$

The equilibrium profit of the firms are

$$\widetilde{\pi}_G^* = \frac{[32t^2 - (t - \Delta/2 + C)^2]^2}{128t^2(t - \Delta/2 + C)}, \text{ and } \widetilde{\pi}_S^* = \frac{(t - \Delta/2 + C)^3}{128t^2}.$$

Next, we compare the equilibrium profits under the two regimes. First, we compare firm G's profit. Let $g(\Delta) = \widetilde{\pi}_G^* - \pi_G^*$. After some algebra, it can be defined as

$$g(\Delta) = \frac{8t^2}{(t - \Delta/2 + C)} + \frac{(t - \Delta/2 + C)^3}{128t^2} - \frac{(t - \Delta/2 + C)}{2} - \frac{(3t + \Delta)^2}{18t} - \frac{t}{\delta}\left[\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2} + \gamma s} - \frac{t}{\delta}\right]. \tag{91}$$

Taking the derivative of the preceding expression w.r.t. Δ gives

$$\frac{\partial g(\Delta)}{\partial \Delta} = \frac{8t^2}{2C(t - \Delta/2 + C)} - \frac{3(t - \Delta/2 + C)^3}{256t^2 C} + \frac{(t - \Delta/2 + C)}{4C} - \frac{(3t + \Delta)}{9t}.$$

As argued in the proof of Proposition 2, the preceding expression is positive for all $0 < \Delta < 3t$. Moreover, $g(0) < 0$ and $g(3t) > 0$. Hence, by intermediate value theorem, there exists $\Delta_G(\delta) \in (0, 3t)$ such that $g(\Delta_G(\delta)) = 0$. In other words, (a) for $0 < \Delta \le \Delta_G(\delta)$, we have $g(\Delta) \le 0$, implying that $\widetilde{\pi}_G^* \le \pi_G^*$, and (b) for $\Delta_G(\delta) < \Delta < 3t$, we have $g(\Delta) > 0$, implying that $\widetilde{\pi}_G^* > \pi_G^*$. Moreover, since π_G^* is strictly decreasing in δ, we have $\Delta_G'(\delta) < 0$.

Next, we compare firm S' profit. Let $s(\Delta) = \widetilde{\pi}_S^* - \pi_S^*$. After some algebra, it can be defined as

$$s(\Delta) = \frac{(t - \Delta/2 + C)^3}{128t^2} - \frac{(3t - \Delta)^2}{18t} - \frac{t}{\delta}\left[\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2} + \gamma s} - \frac{t}{\delta}\right]. \tag{92}$$

Taking the derivative of the preceding expression w.r.t. Δ gives

$$\frac{\partial s(\Delta)}{\partial \Delta} = -\frac{3(t-\Delta/2+C)^3}{256Ct^2} + \frac{(3t-\Delta)}{9t}.$$

As argued in the proof of Proposition 2, $\frac{\partial s(\Delta)}{\partial \Delta} > 0$ for small values of Δ, and $\frac{\partial s(\Delta)}{\partial \Delta} < 0$, for large values of Δ. Moreover, $\frac{\partial^2 s(\Delta)}{\partial \Delta^2} < 0$, thus $s(\Delta)$ is concave in Δ. Since $s(0) < 0$ and $s(3t) > 0$, by intermediate value theorem, there exists $\Delta_S(\delta) \in (0, 3t)$ such that $s(\Delta_S(\delta)) = 0$. In other words, (a) for $0 < \Delta \leq \Delta_S(\delta)$, we have $s(\Delta) \leq 0$ implying that $\widetilde{\pi}_S^* \leq \pi_S^*$ and (b) for $\Delta_S(\delta) < \Delta < 3t$, we have $s(\Delta) > 0$ implying that $\widetilde{\pi}_S^* > \pi_S^*$. Moreover, since π_S^* is strictly decreasing in δ, we have $\Delta_S'(\delta) < 0$.

Finally, we can argue that the likelihood of service integration decreases as γ or s increases. Examining $g(\Delta)$ (defined by Equation (91)) and $s(\Delta)$ (defined by Equation (92)), we can see that both are strictly decreasing in γ and s. Hence, the profit difference decreases as γ or s increases, implying that the thresholds $\Delta_G(\delta)$ and $\Delta_S(\delta)$ would be higher as γ or s increases. This completes the proof. ■